

# On a MEMS-Based Parametrically Amplified Atomic Force Sensor

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## SUMMARY

We have built a functional MEMS-based parametric amplifier. This system amplifies the output of a MEMS-based atomic-scale force sensor. We have separately characterized the two main components (parametric amplifier and atomic-scale force sensor) in addition to characterization of the complete system. The parametric amplifier provides a gain of 49.9 dB. The force sensor displays a frequency shift of 62.4% when the gap is small. When combined into a complete system, the output of the force sensor is amplified by 47.7 dB. Our complete system is fully monolithic, with built-in actuation, displacement detection, bandpass filtering, and nonlinear mixing.

**Keywords:** parametric, amplification, van der Waals

## INTRODUCTION

One of the fascinating aspects of MEMS is the capability to integrate formerly incompatible components into one monolithic system. In this work, we present such an application: integrating an atomic-scale force sensor with a mechanical parametric amplifier. By integrating our parametric amplifier with the sensor, we amplify our signal before it leaves the chip, thereby reducing possible deleterious influences (via increased signal-to-noise ratio) from wirebonds, package pins, and external wiring.

The system we describe is entirely monolithic, including tip actuation, signal amplification, and capacitive displacement detection all on one chip. Everything necessary for the measurement, other than signal sources, was created on one 6 mm by 6 mm chip with the very straightforward fabrication process of SCREAM [1].

A parametric amplifier uses a nonlinear or time-varying component in an otherwise linear system to produce power gain for an input signal. There are two inputs, the *signal* to be amplified and a *pump* signal. The nonlinear component acts as a frequency mixer which creates all of the harmonics of the two input frequencies. If we filter out all of the harmonics except for one (the output frequency), the powers from the input signal and the pump signal will be transferred to this output frequency to achieve net power gain.

An AFM is a device which detects the atomic-scale

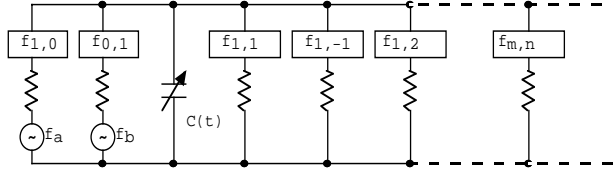


Figure 1: *Manley-Rowe circuit model.*

forces including the van der Waals (vdW) forces between two bodies. AFM systems are sensitive to small variations of displacement, and are capable of atomic-scale imaging of surfaces.

## PARAMETRIC AMPLIFICATION

It can be shown that for the purposes of parametric amplification, a time-varying reactance is equivalent to a nonlinear reactance [2]. Manley and Rowe provide the framework for understanding the power flow relationships when using a system with such a reactance. They provide criteria for determining whether or not power gain is possible and what the maximum gain may be under ideal circumstances.

Figure 1 shows the generalized system that Manley and Rowe analyzed. Note, there are two input signals at frequencies  $f_a$  and  $f_b$ . There are also bandpass filters and associated resistances at those frequencies, designed to reject power not within the bandpass (i.e. they do not resistively dissipate the energy of unwanted frequencies). In addition to these inputs and the reactance (shown here as a time-dependent capacitor), there is an infinite array of load resistances and associated bandpass filters attached to the system. The frequencies of these additional filter/load pairs are located at all of the sums and differences of the two input frequencies. The symbolic convention we will use is  $f_{m,n}$ , where the first subscript is the number  $m$  times  $f_a$ , and the second subscript is the number  $n$  times  $f_b$ . This means that  $f_{1,0} = f_a$  and  $f_{1,2} = f_a + 2 \times f_b$ .

The sign convention we will use is that power flowing into the reactance (from the sources) is positive, and power flowing from the reactance (into the loads) is negative. If we were to remove *all* but the  $f_{1,1}$  load, this would reduce the Manley-Rowe equations to:

$$\frac{P_{1,0}}{f_{1,0}} + \frac{P_{1,1}}{f_{1,1}} = 0 \quad \text{and} \quad \frac{P_{0,1}}{f_{0,1}} + \frac{P_{1,1}}{f_{1,1}} = 0 \quad (1)$$

In words, we are supplying  $P_{1,0}$  and  $P_{0,1}$  to the reactance from the input sources. This means  $P_{1,1}$  must be negative and power flows from the reactance into the load at  $f_{1,1}$ . We define the power gain,  $g_{m,n}$ , of this system:

$$g_{1,0} = \frac{-P_{1,1}}{P_{1,0}} = \frac{f_{1,1}}{f_{1,0}} \quad \text{and} \quad g_{0,1} = \frac{-P_{1,1}}{P_{0,1}} = \frac{f_{1,1}}{f_{0,1}} \quad (2)$$

This particular system is called an up-converter (also called a non-inverting amplifier). We can work out the same set of equations for any similar system. The gain calculated by this method is the maximum theoretical gain achievable in any particular configuration. In reality the gain is limited by unaccounted for reactances, resistive losses, imperfect filters, and imperfect mixing.

## Design of a MEMS Parametric Amplifier

Experimentally, we have determined that the  $Q$  of a SCREAM device is on the order of 1000 at a pressure of 3 mT. A  $Q$  of about 1000 is sufficient for proof-of-concept parametric amplifiers. We would, however, gain by increasing the  $Q$  of our resonators.

A high resonant frequency for the output filter allows for larger gains (due to Equation 2) at the cost of either significantly reduced mass or dramatically increased spring constant. A high spring constant is unacceptable, as it will restrict the maximum displacement available, which makes displacement detection more difficult. Too small of a mass is also a problem.

We will use a double-folded design for our springs and comb drives for our sensors and actuators. We still need a time-dependent reactance to make our system complete. Using the work done [3], we will need to add a time-dependent term to either the zeroth or the second order terms of the MEMS's equation of motion. Since we cannot fabricate a simple time-dependent mass using MEMS techniques, we use a variable stiffness:

$$m\ddot{x} + c\dot{x} + (k + k_{electrical})x = F_{excite}(V_e) \quad (3)$$

where  $m$  is the lumped mass of our system,  $x$  is the displacement,  $c$  is the damping coefficient,  $k$  is the lumped mechanical stiffness of our springs,  $k_{electrical}$  is the lumped electrostatic stiffness, and  $F_{excite}(V_e)$  is the excitation force on the system as a whole. Here, the excitation force is applied by one of the comb drives. If we consider a parallel plate drive for the electrostatic stiffness (see [3]), and perform a Taylor expansion about the stable point,  $x_{\ell\ell}$ , we find the electrostatic stiffness:

$$k_{electrical} = -\frac{\epsilon_0 A V_t^2}{(d - x_{\ell\ell})^3} \quad (4)$$

where  $A$  is the area of the plates,  $V_t$  is the voltage applied across the plates, and  $d$  is the initial gap between plates.

Our device has a resonant frequency of approximately 6 kHz. The device has 132 fingers in the signal bank of

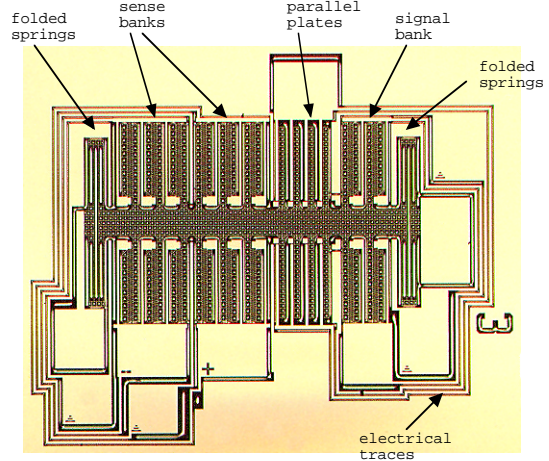


Figure 2: Overview of our device, as viewed from an optical microscope.

comb drives, 198 fingers each on the sense banks of comb drives, all with an initial gap of  $3.5 \mu\text{m}$ . The springs are  $250 \mu\text{m}$  long and  $1 \mu\text{m}$  wide. The 8 parallel plate drives are each  $300 \mu\text{m}$  long at an initial separation of  $4 \mu\text{m}$ . The measured depth of this device is  $30.7 \mu\text{m}$  which makes the total parallel plate area  $73680 \mu\text{m}^2$ . The design, as tested in the next section, is shown in Figure 2.

## Measurements of a Parametric Amplifier

To demonstrate a working MEMS parametric amplifier, we must show three things: that the Manley-Rowe equations hold, that our system has gain, and that the output amplitude is linearly related to the input amplitude. The most direct method of demonstrating that the Manley-Rowe power relationships hold is to plot the gain versus the ratio of frequencies. From the discussion above, the gain is proportional to the ratio of frequencies.

It is impractical to attempt this measurement directly, as our system has a  $Q$  of about 1000. Any attempt to characterize the gain vs. the ratio of the frequencies would only be applicable over a very small frequency range unless we deconvolve it from the bandpass behavior of the mechanical device. We will bypass the issue of bandpass filtering and perform this measurement away from resonance. Thus, our system would look like the one in Figure 1 wherein all of the harmonics are present across the time-varying reactance. This leads to the undesirable consequence of negative gain (i.e. attenuation) due to the power being distributed among the other harmonics. Nonetheless, we still observe the behavior expected of the Manley-Rowe equations, as seen in Figure 3.

As there is attenuation, we have normalized the measured behavior to the theoretical, which is simply a plot of  $\frac{f_{-1,1}}{f_{1,0}}$  versus  $f_{1,0}$ . We have found this frequency dependent gain to occur over a wide parameter space.

Figure 4 shows gain over a limited parameter space.

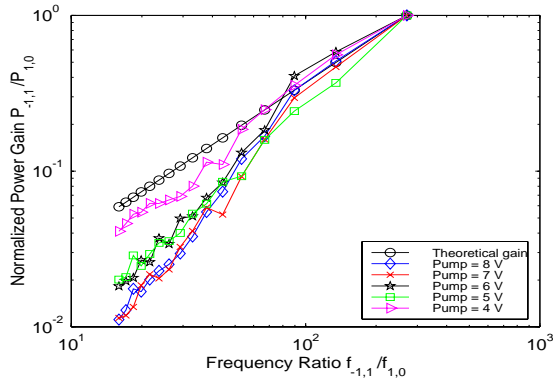


Figure 3: Gain vs. the ratio of frequencies,  $f_{0,1} = 5$  kHz at five voltages,  $f_{1,0} = 20$  Hz to 350 Hz.

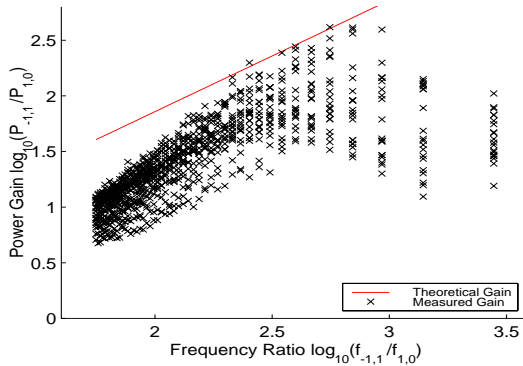


Figure 4: Large gain vs. the ratio of frequencies,  $f_{0,1} = 5.7$  kHz from 4 to 8 V, with a DC offset of 3 V,  $f_{1,0} = 0$  Hz to 100 Hz at 10 V p-p.

The theoretical curve in Figure 4 is normalized to the maximum gain of 316.2. Figure 5 shows that the gain is constant regardless of the input signal amplitude at several input frequencies.

## ATOMIC FORCE SENSING WITH PARAMETRIC AMPLIFICATION

In this section we will integrate the parametric amplifier with a MEMS-based sensor.

The device used in this section is described fully in Chapter 2 of [4]. It is a parametric amplifier, similar to the design in the previous section, coupled via a mechanical spring to an atomic-scale force sensor (AFM).

The frequency of our sensor shifts due to the van der Waals force as a strong function of interaction distance. If we were to rigidly attach the parametric amplifier to the AFM, then there would be energy fed back into the sensor. This would produce a closed loop wherein the very act of amplification would change the measurement.

We therefore move our amplifier to a separate resonator with a higher resonant frequency than the AFM sensor structure. The sensor structure is loosely coupled to the amplifier structure with a mechanical cou-

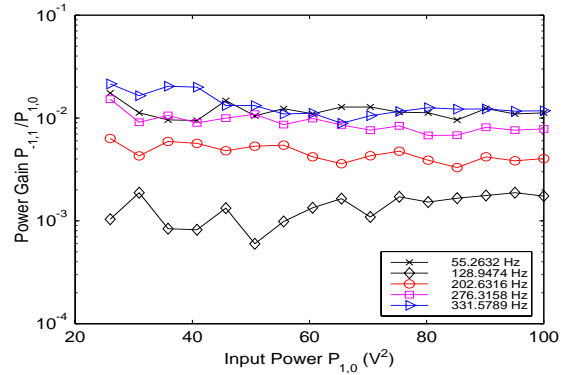


Figure 5: Gain vs. input amplitude,  $f_{1,0}$  at 5 frequencies from 5 to 10 V p-p,  $f_{0,1}$  is 8 V p-p at 10 kHz.

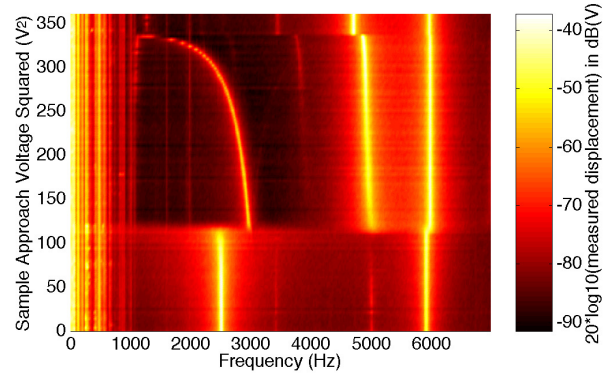


Figure 6: Characterization of combined AFM/paramp device. The tip is driven by a pseudo-random signal at 5 V between 1 and 3 kHz.

pling spring. Here, the low frequency AFM measurement transfers power efficiently to the parametric amplifier, but the high frequency pump and output signals will not be transferred back to the sensor.

As discussed above we selected a resonant frequency of about 6 kHz for the amplifier portion of the system. The resonant frequency for the sensor portion of the system is approximately 3 kHz.

## Results/Discussion

Figure 6 shows the behavior of the AFM tip interacting with the sample without the benefit of parametric amplification. The 2496 Hz peak is the resonant frequency of the AFM structure when it is not interacting with the sample. When the sample approach actuator is at 10.63 V (113 V<sup>2</sup>), the tip enters a bi-stable state between interacting and not interacting. At higher sample approach voltages (i.e. when the tip is closer to the sample), the tip is interacting with the sample and we observe a frequency shifting behavior characteristic of these sorts of systems. When the sample approach voltage is about 18.4 V (339 V<sup>2</sup>), the tip and sample come into contact.

Once we apply the pump to the amplifier, things start getting interesting. Figure 7 shows the resulting behavior. Note how the pull-in behavior of the tip is mixed to a

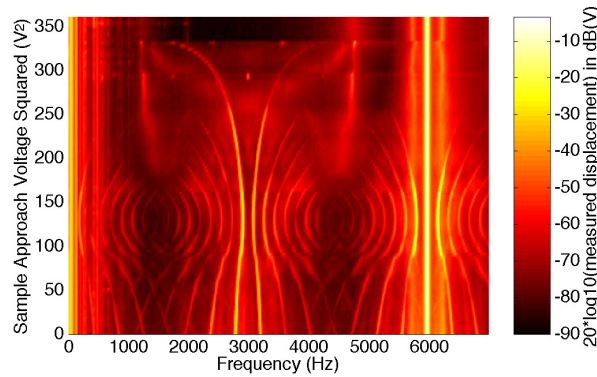


Figure 7: *Parametrically amplified AFM measurement. The conditions are the same as in Figure 6 except the parallel plate actuators are driven at 3 V p-p at 5976 Hz with a 1.5 V DC offset.*

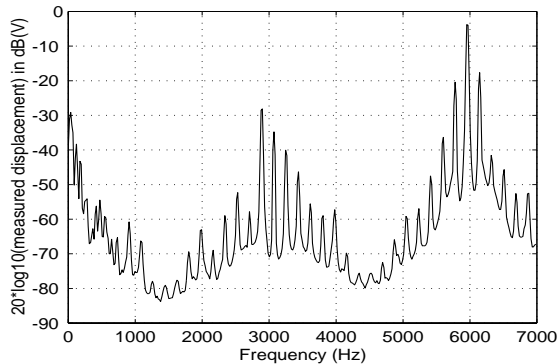


Figure 8: *Cross-section of Figure 7 showing the frequency response at the point of highest gain. The sample approach voltage is 10.8 V (117 V²).*

variety of frequencies. Also note that the mixed signals near the resonant frequency of the amplifier mass have the highest amplitudes.

Figure 8 shows a “cross-section” (i.e. the frequency response) of Figure 7 at the point of highest gain. Here, we see that the amplitude of the input signal from the tip mass is -28.1 dBV at 2898 Hz and the output signal is -17.65 dBV at 6145 Hz giving us a gain of 10.45 dB.

If we increase the pumping to 7 V with a 3.5 V DC offset and reduce its frequency to 5500 Hz, we get a gain of 47.75 dB, well over two orders of magnitude stronger.

The noise floor of the system during sample approach under non-amplified conditions is about -87 dBV (from Figure 6). This means we have increased our signal-to-noise ratio from roughly 33:1 to 2867:1, so we can drastically reduce our pseudo-random tip driving signal or even eliminate it altogether and rely on thermomechanical noise to generate the resonance peak at the tip.

## CONCLUSION

We have presented an electromechanical parametric amplifier with built-in displacement detection and band-

pass filtering where the time-varying component is an electrostatic spring constant. Although prior work has been performed using one or more of these characteristics, there is no record of any such system incorporating all of them or with nearly as much gain [5], and [6]. One group has built a degenerate parametrically amplified microcantilever, with a gain of slightly less than 20 dB and a non-degenerate gain of 1 dB [7]. Another group has built a parametrically amplified membrane (a MEMS tuned capacitor), with an up-converter gain of 6 dB [8].

We have also presented an atomic-scale force sensor and attached it to another electromechanical parametric amplifier. The gain of this parametrically amplified atomic force measurement is 47.75 dB. Since this sort of parametric amplification is purely mechanical, we could attach it to just about any other MEMS sensor to provide on-chip pre-amplification.

Also, since the signal is amplified *without the use of transistors*, this gives us the flexibility of choosing a MEMS fabrication process that is not limited by the requirements of VLSI technology.

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