On A MEMS-Based Parametrically Amplified Atomic Force Sensor

Michael Wolfson
Cornell University

“B” Exam
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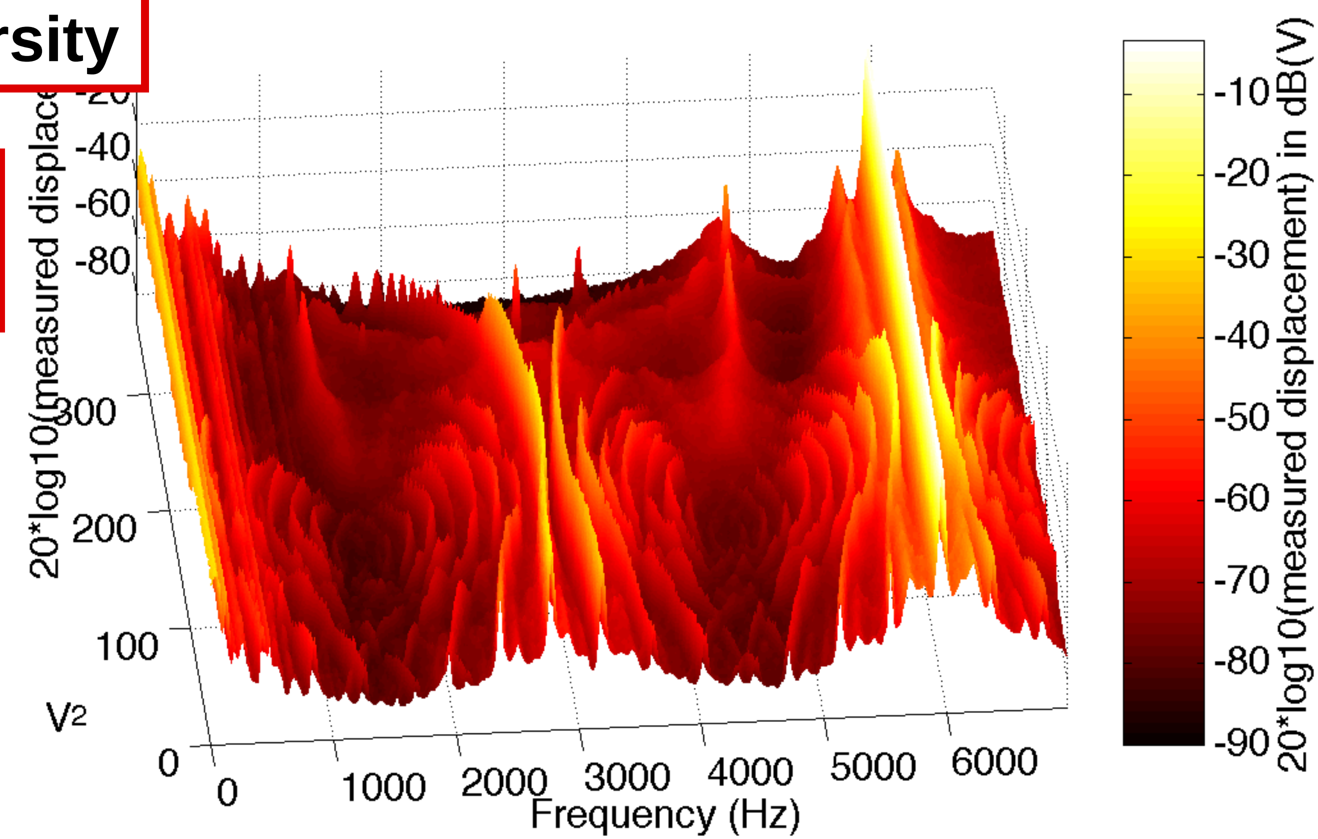


Outline

- **Parametric amplifier theory**
  - **Manley-Rowe power equations**
- **Test environment**
  - **Equipment**
  - **MEMS process**
- **Parametric Amplifier experiment**
  - **MEMS design**
  - **Results**
- **AFM Experiment**
  - **MEMS design**
  - **Results**
- **Combined Experiment**
  - **Results**
- **Conclusion**

Parametric Amplification Motivation

On-chip preamplification of signals

- Amplifier integrated with sensor
- Reduce effect of parasitic capacitance
- No transistors
- No need to integrate VLSI with MEMS
- Low noise gain
- Upconvert to a higher frequency

Parametric Amplification Premise

To use nonlinear mixing to raise small, low frequency signals above the noise floor

- **Using a characterized nonlinear component in an otherwise linear system, we can convolve a small amplitude, low frequency signal (signal) with a high frequency, high amplitude signal (“carrier” or “pump”)**

- **This is a subset of a phenomenon known as ‘mixing’, wherein the convolution of two signals produces the set of all of the harmonics.**

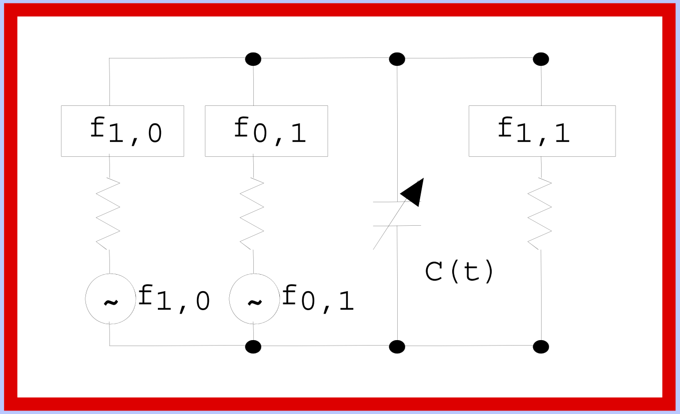
- **If, at the output, we filter all but one of the harmonics, then we can demodulate this signal to restore the initial input.**

Parametric Amplification

If certain criteria are met, then the system will function as a *parametric amplifier.*

These conditions are:

- **Single-valued, lossless, non-linear device**
- **Weak coupling between modes in non-linear device**
- **Perfect filters (i.e. non-resonant frequencies see an open circuit)**
- **Circuit layout as such:**



If these conditions hold, then the Manley-Rowe equations reduce to:

$$\frac{P_{0,1}}{f_{0,1}} + \frac{P_{1,1}}{f_{1,1}} = 0 \quad \textbf{and} \quad \frac{P_{1,0}}{f_{1,0}} + \frac{P_{1,1}}{f_{1,1}} = 0$$

where gain ($P_{1,1}/P_{1,0}$) depends only on the ratio of frequencies

Amplitude Modulation

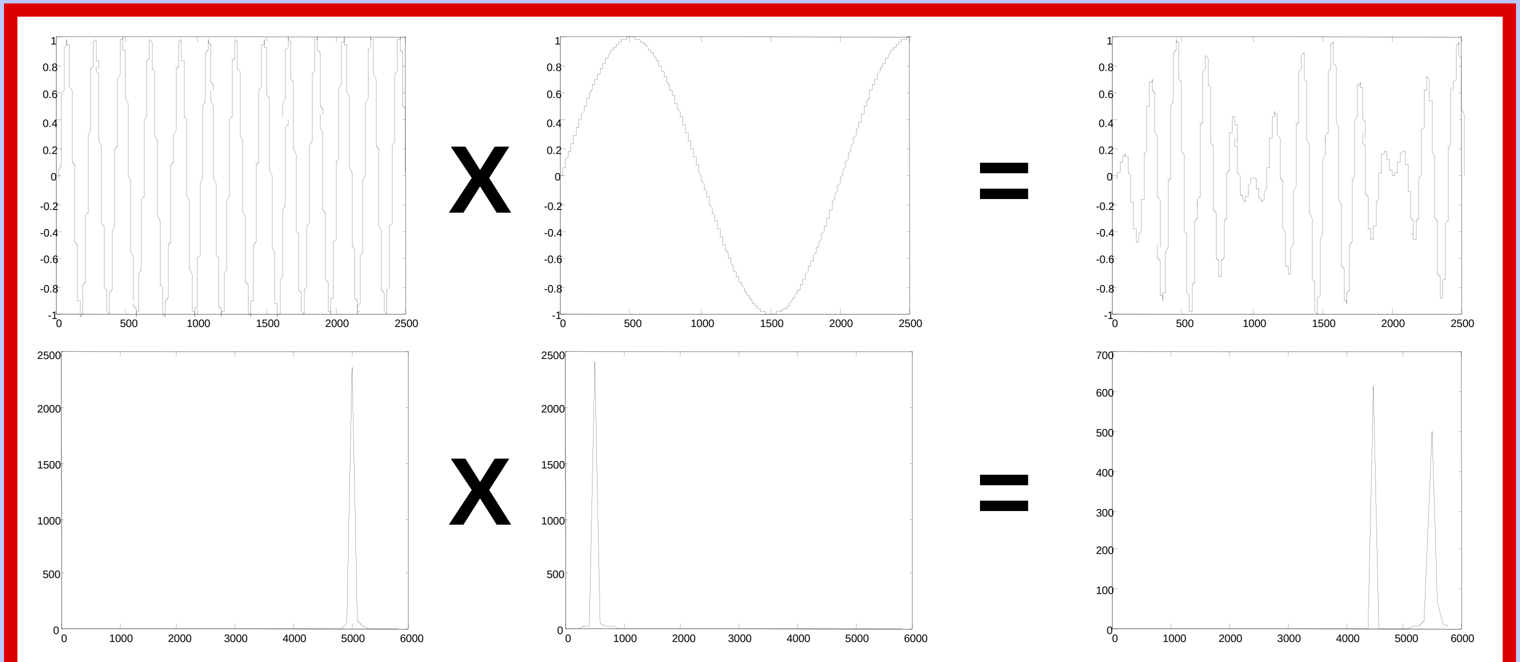
Given arbitrary modulating signal, f(t), and a carrier, cos(ω_0 t), we convolve to get the modulated signal:

$$f(t) \Leftrightarrow F(\omega)$$

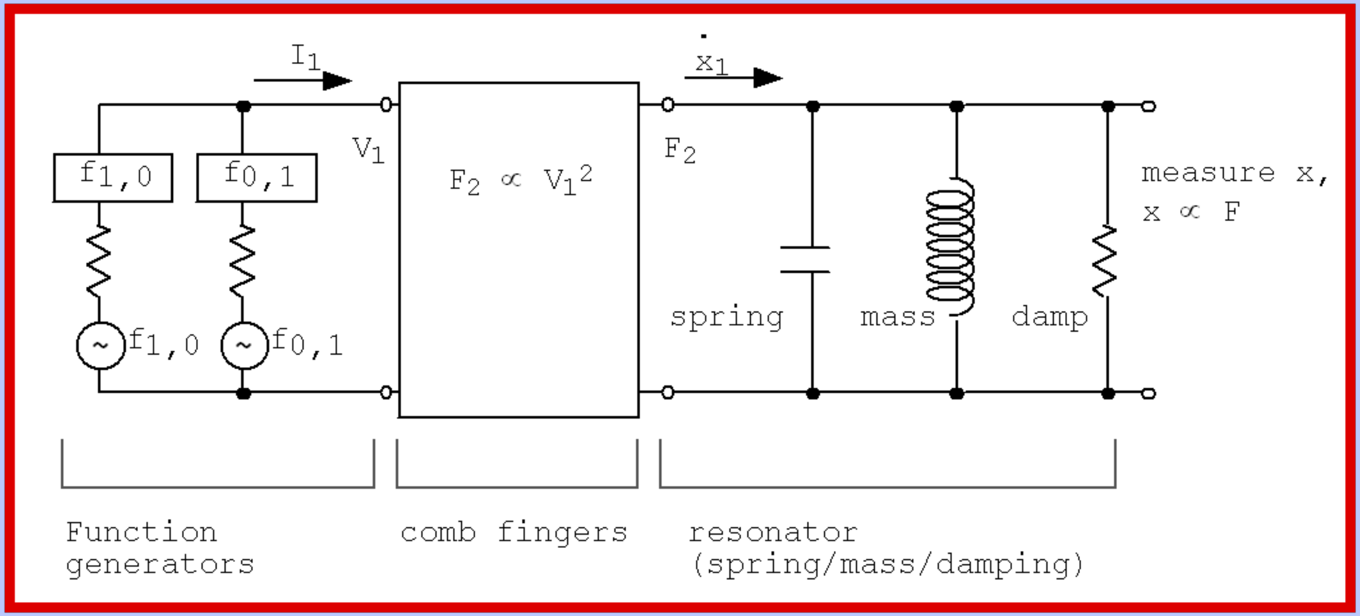
$$g(t) = \cos(\omega_0 t) = \tfrac{1}{2}\left(e^{j\omega_0 t} + e^{j\omega_0 t}\right) \Leftrightarrow G(\omega) = \pi\left[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)\right]$$

$$f(t)\cos(\omega_0 t) \Leftrightarrow F(\omega) * G(\omega) = \tfrac{\pi}{2}\left[F(\omega - \omega_0) + F(\omega + \omega_0)\right]$$

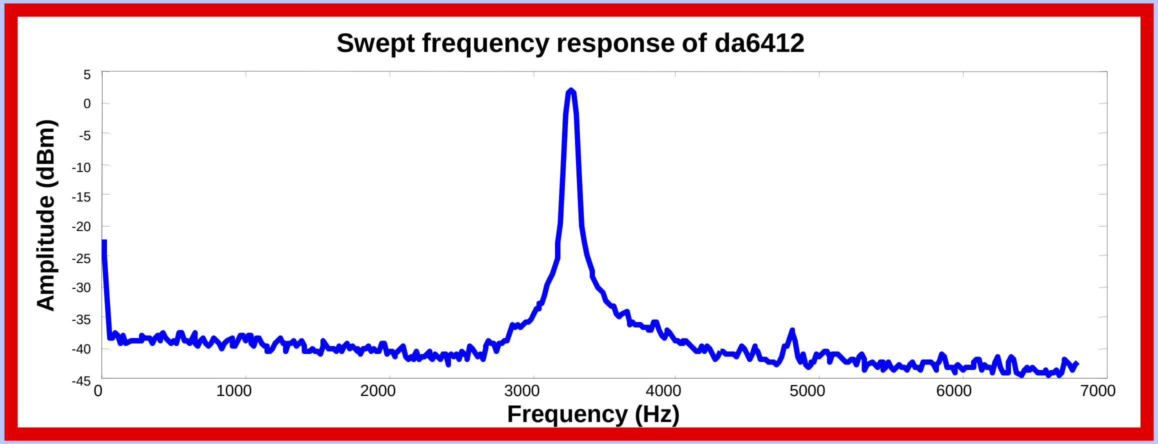
- **Time**
- **Freq**



The System Being Tested



Using only one function generator, we can find the frequency response characteristics of the resonator system:



Operation as a Mixer

According to the previous schematic, the force applied to the resonator (and thus the displacement) is proportional to the input voltage squared.

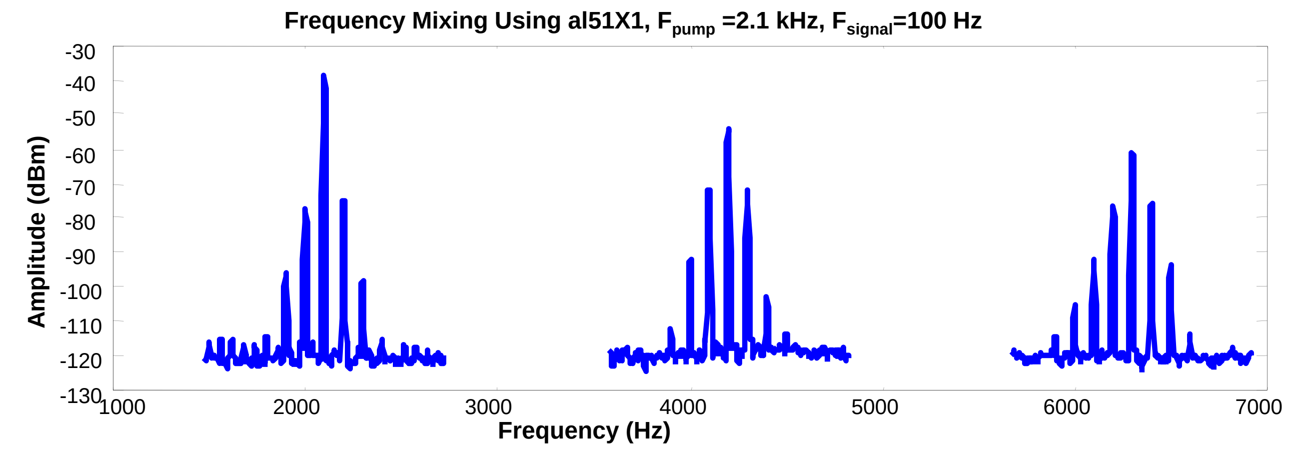
If we have two sine waves in V, this leads to:

$$F \propto \Big(\big(V_{a0} + V_a \sin(\omega_a t)\big) + \big(V_{b0} + V_b \sin(\omega_b t)\big)\Big)^2 \qquad V_{c0} = V_{a0} + V_{b0}$$

$$F \propto V_{c0}{}^2 + 2V_{c0}V_a \sin(\omega_a t) + 2V_{c0}V_a \sin(\omega_a t) + 2V_a \sin(\omega_a t)V_b \sin(\omega_b t) + \ldots$$

$$\big(V_a \sin(\omega_a t)\big)^2 + \big(V_b \sin(\omega_b t)\big)^2$$

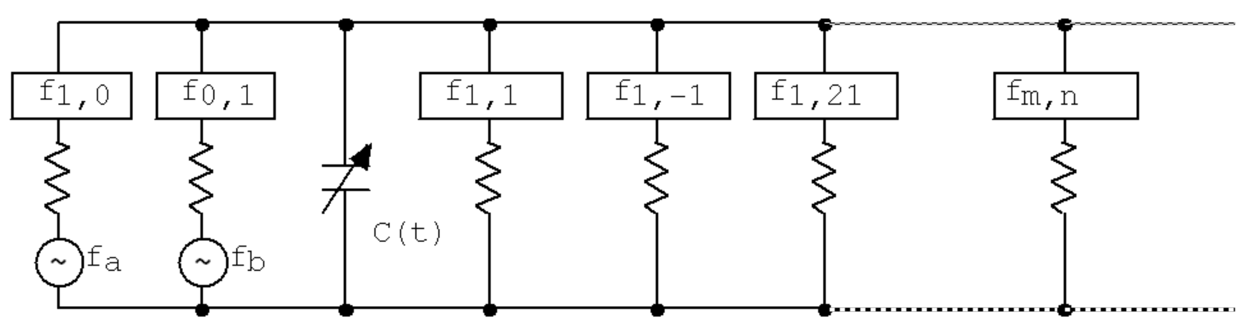
Recall, multiplication in time domain is equivalent to convolution in frequency domain.



Manley-Rowe Power Equations

Assumptions/Terminology:

- **Non-linear capacitor, no hysteresis, voltage is a single-valued function of q,** $v(q)$
- $f_{m,n} = mf_1 + nf_2$
- $q(t) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} Q_{m,n} \exp[j(m\omega_1 t + n\omega_2 t)]$
- **For Real q,**



Manley-Rowe Power Equations

$$i = \frac{dq}{dt} = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} j\left(m\omega_1 t + n\omega_2 t\right) Q_{m,n} \exp[\, j\left(m\omega_1 t + n\omega_2 t\right)]$$

$$v = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} V_{m,n} \exp[\, j\left(m\omega_1 t + n\omega_2 t\right)]$$

Find Fourier coefficients:

$$V_{m,n} = \tfrac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} v(q) \exp[-j\left(m\omega_1 t + n\omega_2 t\right)] \; d(\omega_1 t) d\left(\omega_2 t\right)$$

$$I_{m,n} = j\left(m\omega_1 t + n\omega_2 t\right) Q_{m,n}$$

Multiply $jmQ^*_{m,n}$ **and** $V_{m,n}$ **and sum over all m,n to find power:**

$$Power = 2\Re\left(VI^*\right)$$

$$\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{mV_{m,n} I^*_{m,n}}{mf_1 + nf_2} = \tfrac{1}{2\pi} \int_0^{2\pi} \int_0^{2\pi} v \tfrac{\partial q}{\partial \omega_1 t} \;\; d(\omega_1 t) d\left(\omega_2 t\right)$$

Manley-Rowe Power Equations

Since q is single-valued and 2 π periodic in ω_1t and ω_2t, the RHS of the previous equation equals zero.

This gives us the Manley-Rowe Power Relation for ω_1:

$$\sum_{m=0}^{\infty}\sum_{n=-\infty}^{\infty}\frac{mP_{m,n}}{mf_1+nf_2}=0$$

Similarly if we follow through with ω_2, then we get:

$$\sum_{m=-\infty}^{\infty}\sum_{n=0}^{\infty}\frac{nP_{m,n}}{mf_1+nf_2}=0$$

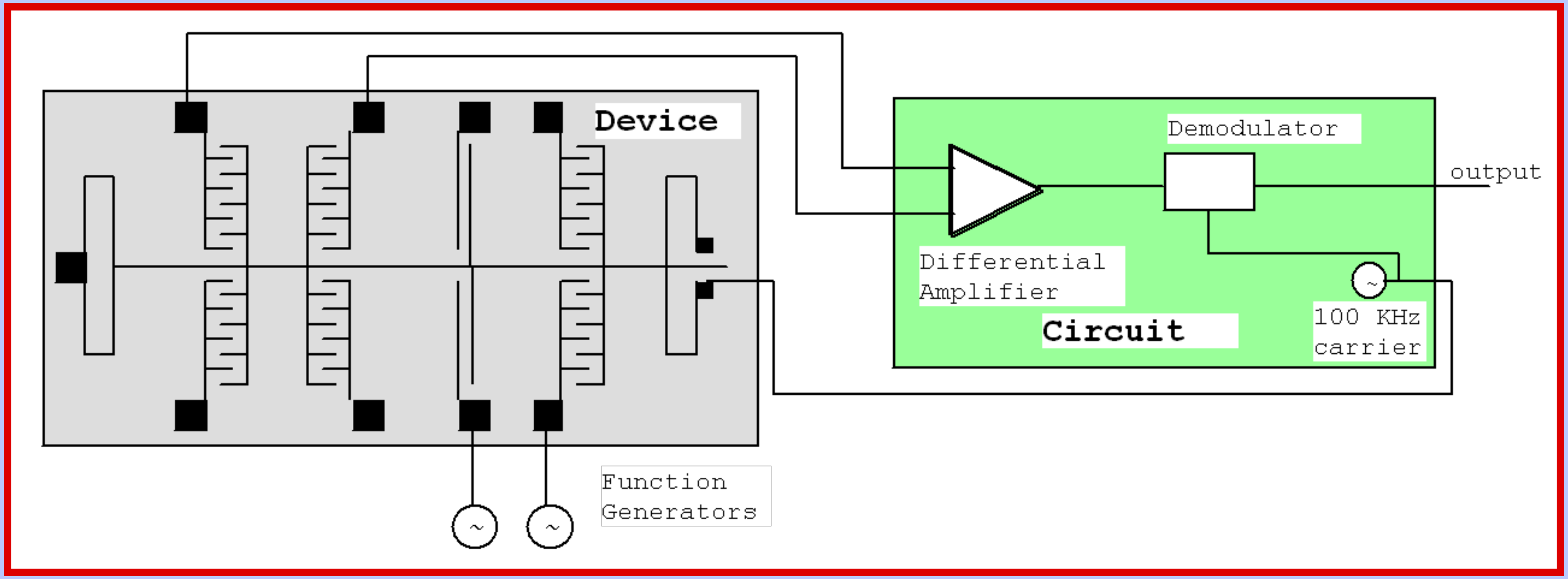
Note, there are very few assumptions involved in this derivation, and it can be applied to other non-linear devices (inductors, mechanical elements).

Also note that the Manley-Rowe power equations are:

- **Independent of power level**
- **Independent of load impedance**
- **Independent of non-linear function**
- **Not limited to small-signal analysis**

Measurement Techniques

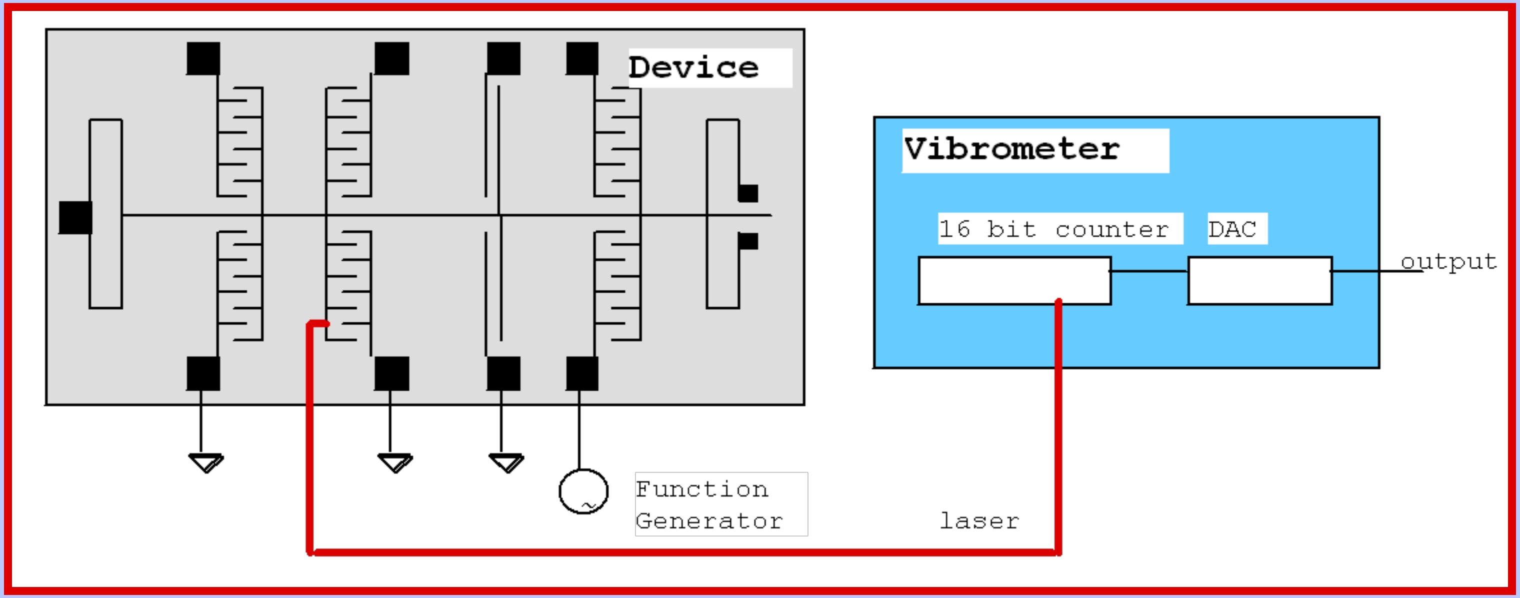
The state (i.e. displacement) of the system was detected using a capacitive sensor via amplitude modulation techniques (carrier frequency = 100 kHz, bandwidth = 8.6 kHz).



All leads are coaxial/shielded from the pins of the differential amplifier to the pins of the packaged device to reduce parasitic capacitance/ambient effects.

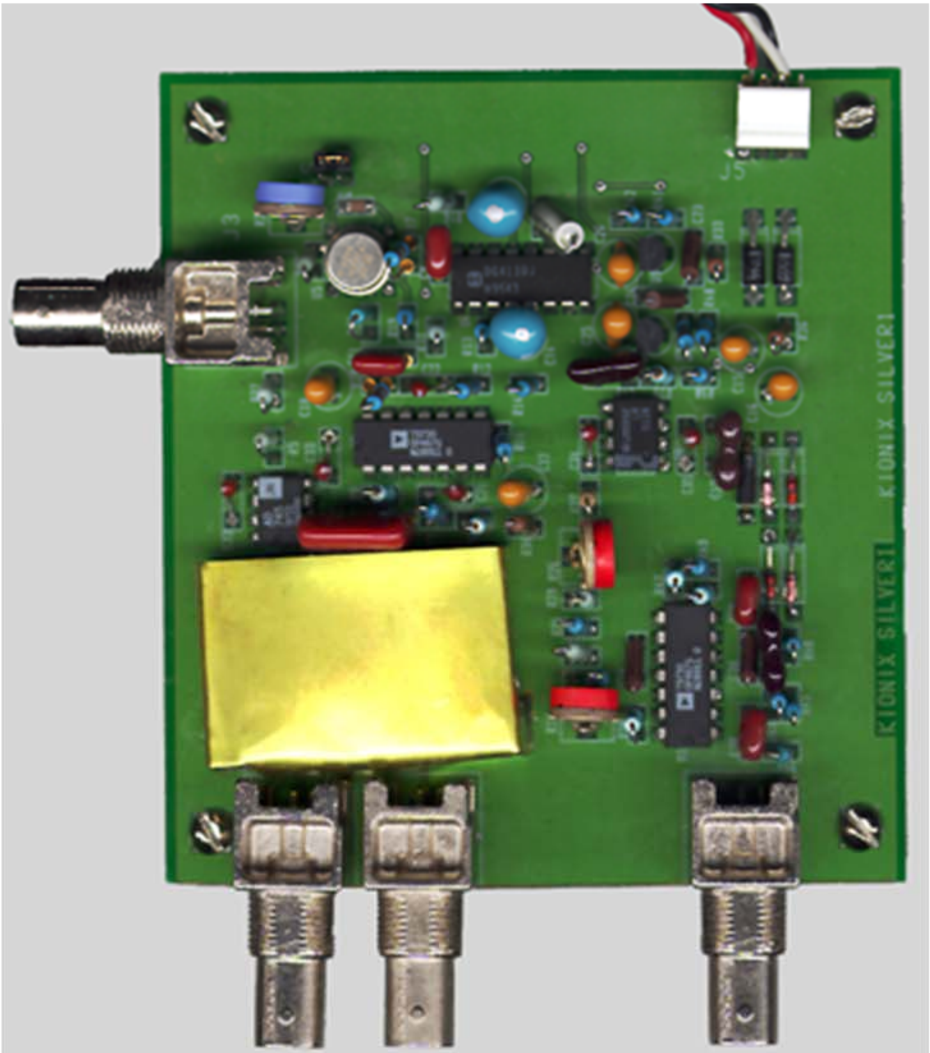
Measurement Techniques

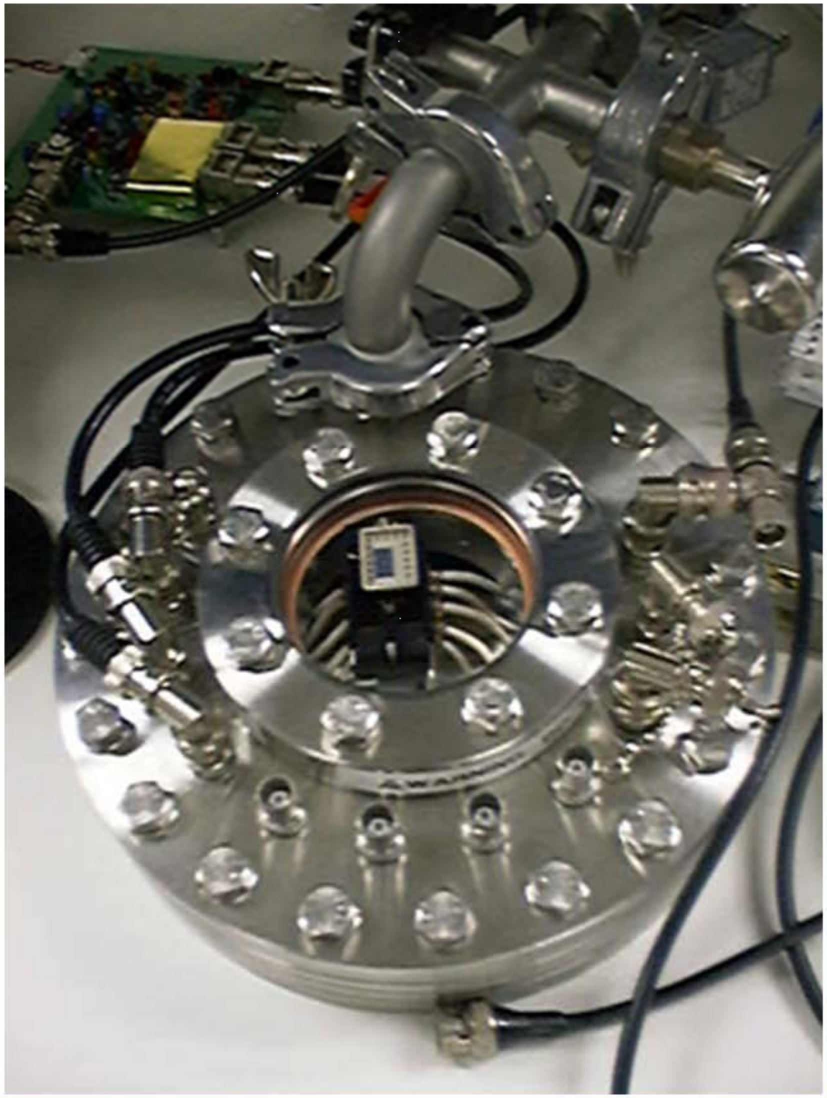
The noise floor of the circuit corresponds to ~0.9 nm, as calibrated by a Polytec OFV3001 Laser Vibrometer system.



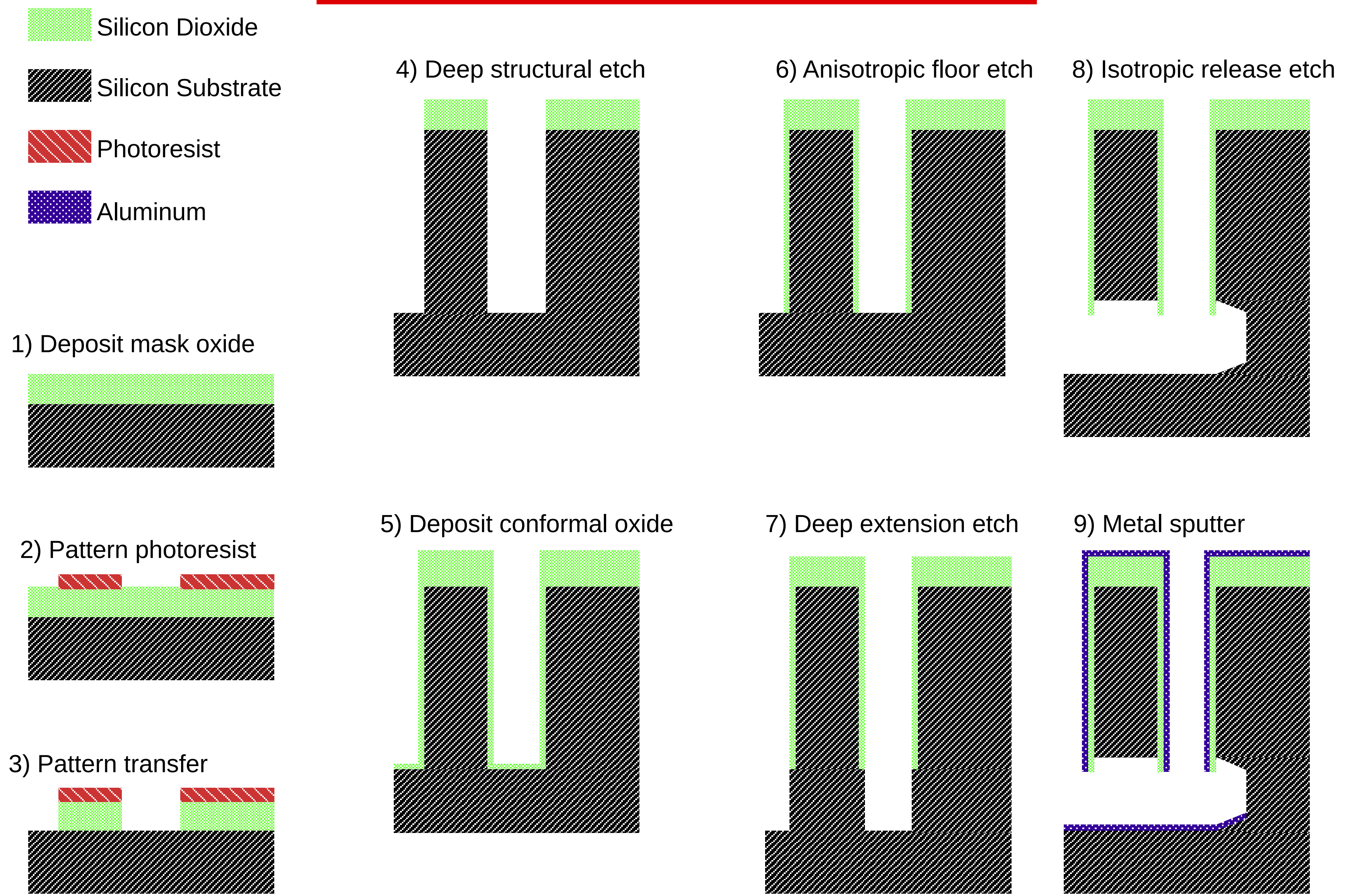
The Laser system was also used to independently verify most measurements.

The Test Setup





MEMS Fabrication

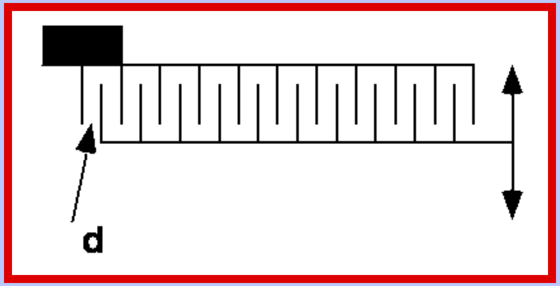


MEMS Design Components

We build our MEMS from a standard toolkit of components which serve particular functions.

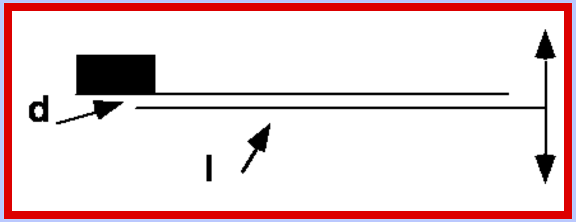
The comb drive is a linear actuator, its force is not a function of the displacement

$$F(V) = \frac{n\varepsilon_0 V^2 h}{d}$$



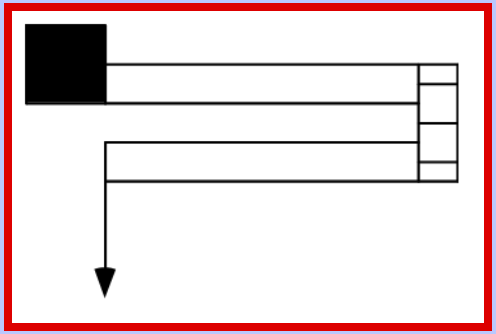
The parallel plate drive is a nonlinear actuator

$$F(V) = \tfrac{1}{2}\varepsilon_0 h \ell \left(\frac{V}{d-x}\right)^2$$

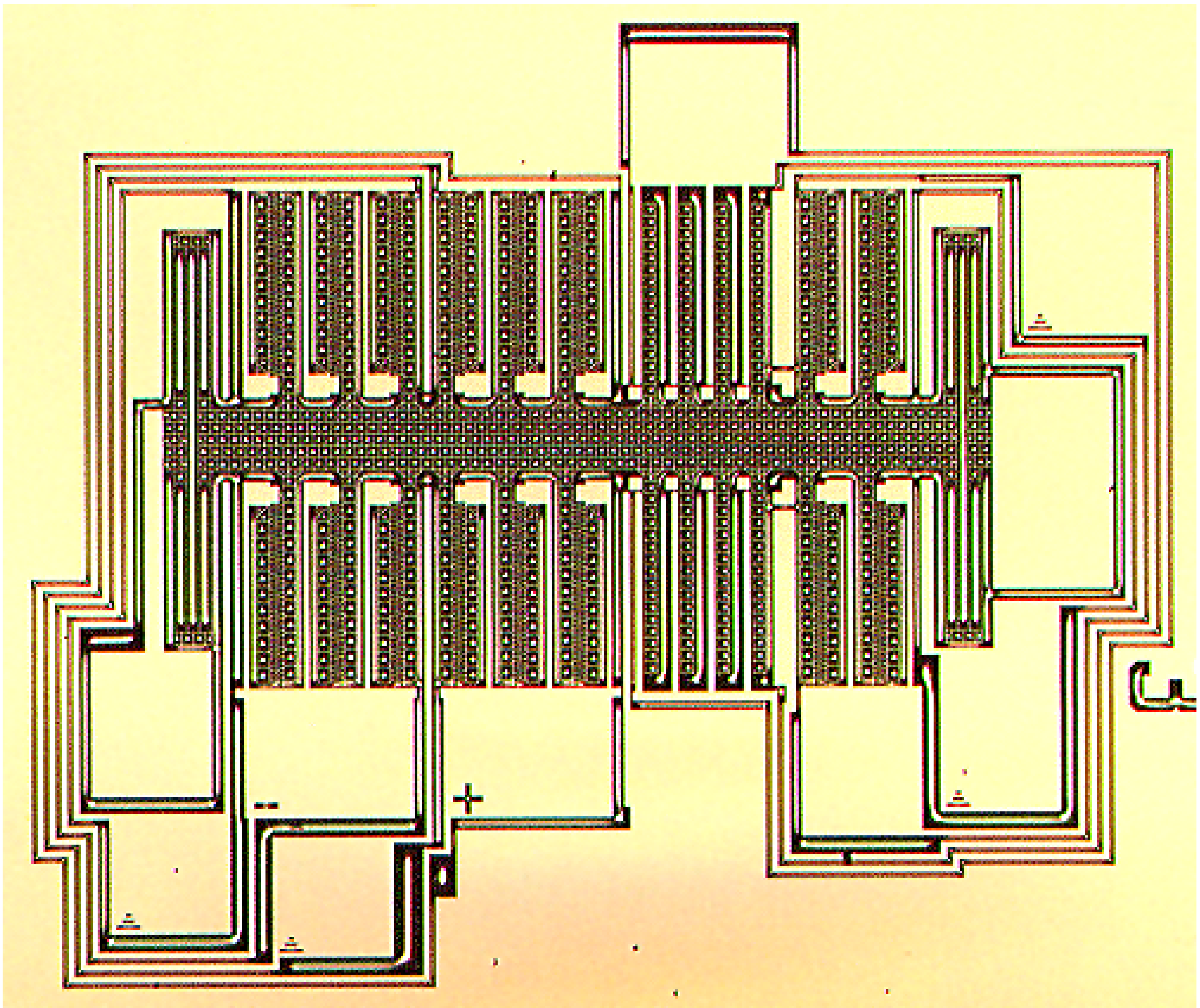


The folded spring is a linear spring

$$F(x) = -kx$$



MEMS Design for paramp

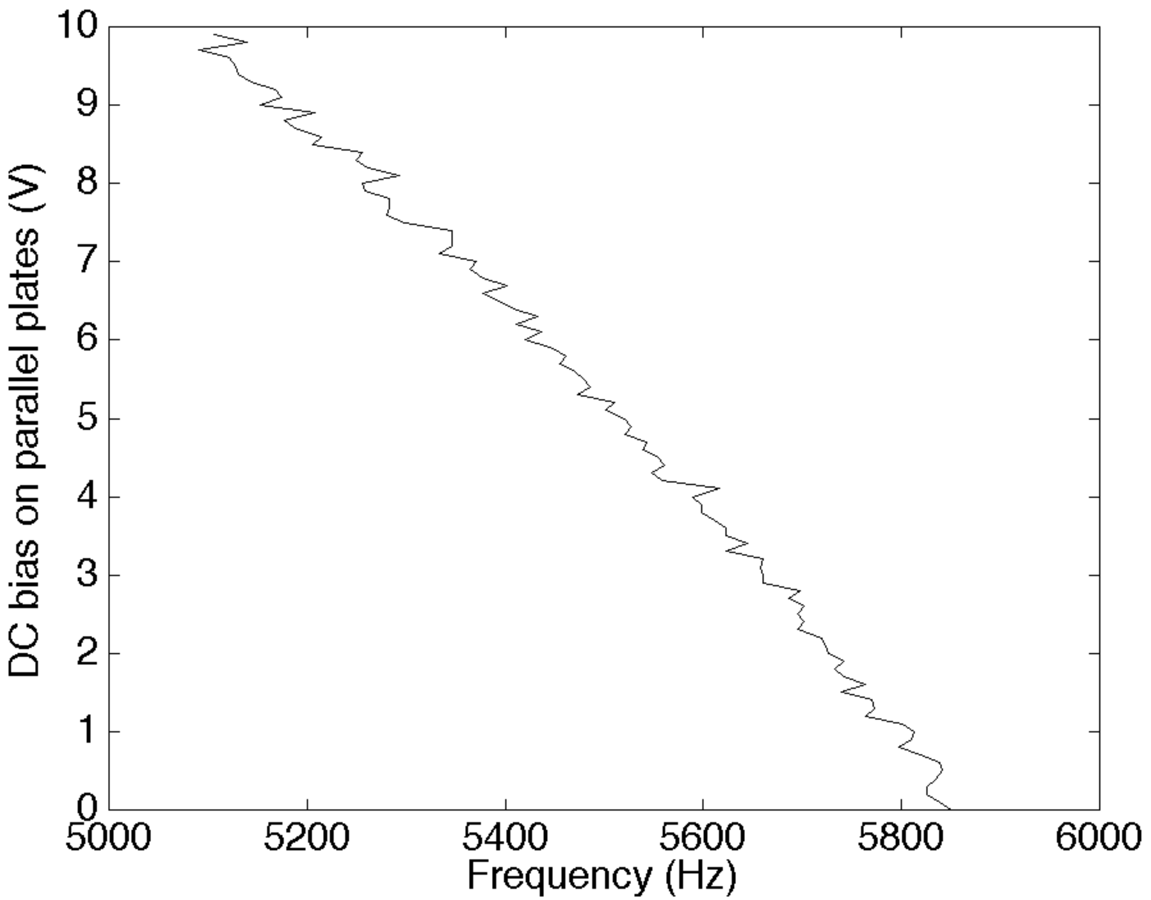


Nonlinearity Characterization

Here, we see that applying a bias on the parallel plate drives does indeed serve to modify the behavior of the system.

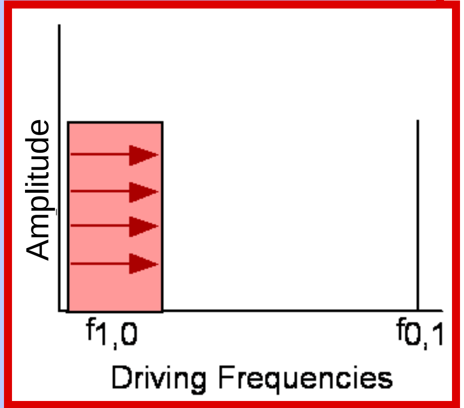
Input is pseudo-random signal from HP 89410 VSA at 4V

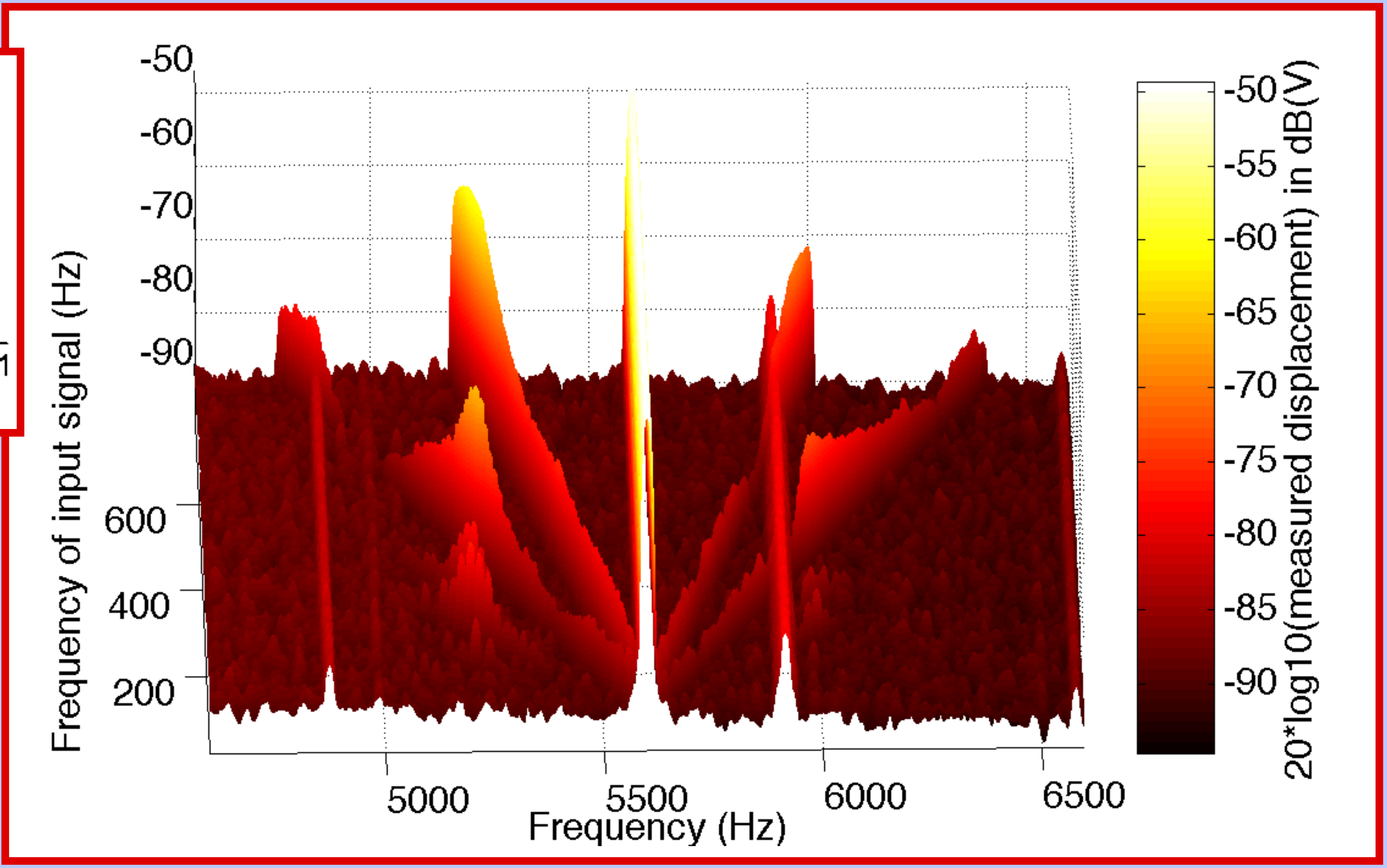
Only maximum at each point is shown (i.e. resonant freq)



Paramp Characterization

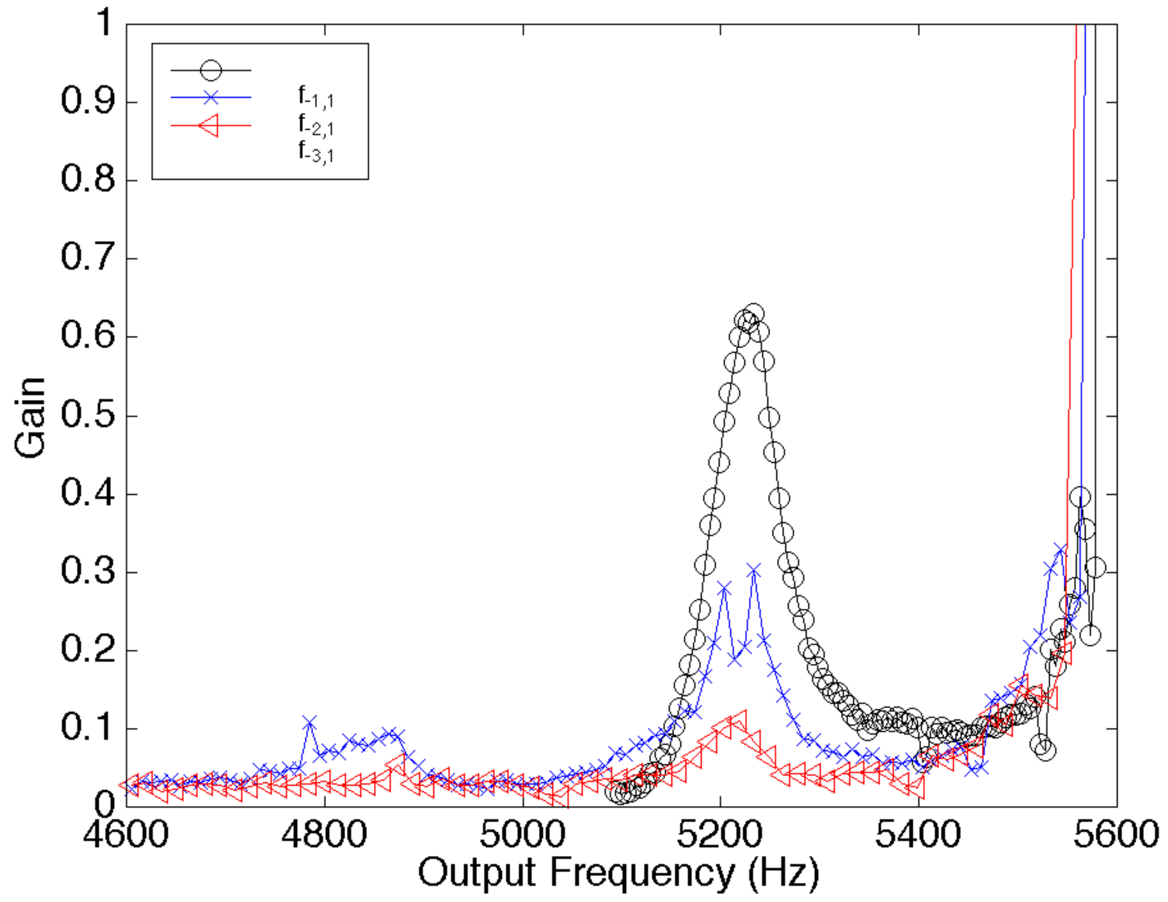
Here, we demonstrate that the output amplitude depends on the impedance in the system (impedance is least at resonance, 5.91 kHz).
The system is pumped by an 8V p-p signal at 5.6 kHz
The input signal is ramped from 10 to 400 Hz at 8 V p-p





Paramp Characterization

Here, we demonstrate that the gain depends on the impedance in the system (impedance is least at resonance, 5.91 kHz). This data is extracted from the previous plot (the same conditions apply)



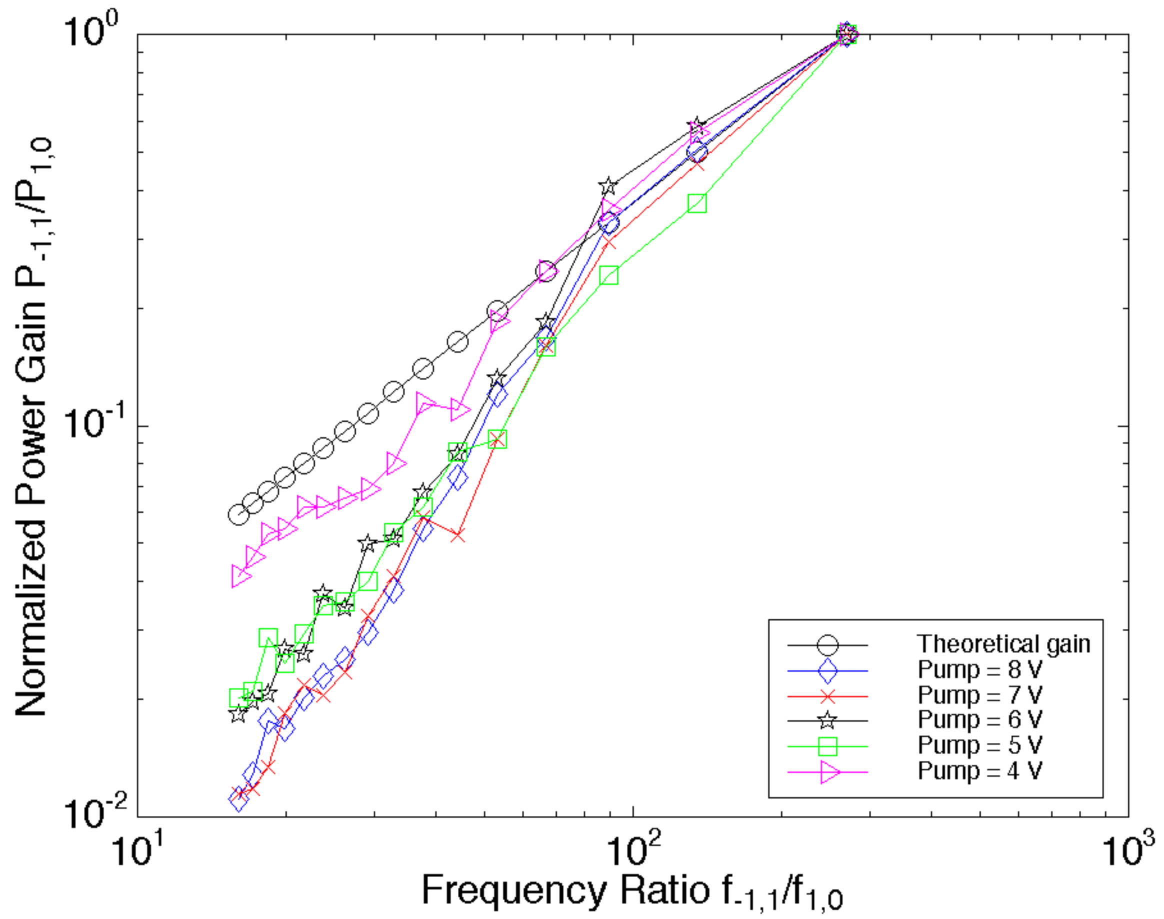
Paramp Characterization

Here, we demonstrate that the gain observes the Manley-Rowe power relationship. That is, the gain is proportional to the ratio of frequencies

Pump at 5 kHz

Signal ramped from 20 to 350 Hz averaged over input amplitudes of 2.5 V to 10 V p-p

Away from resonance



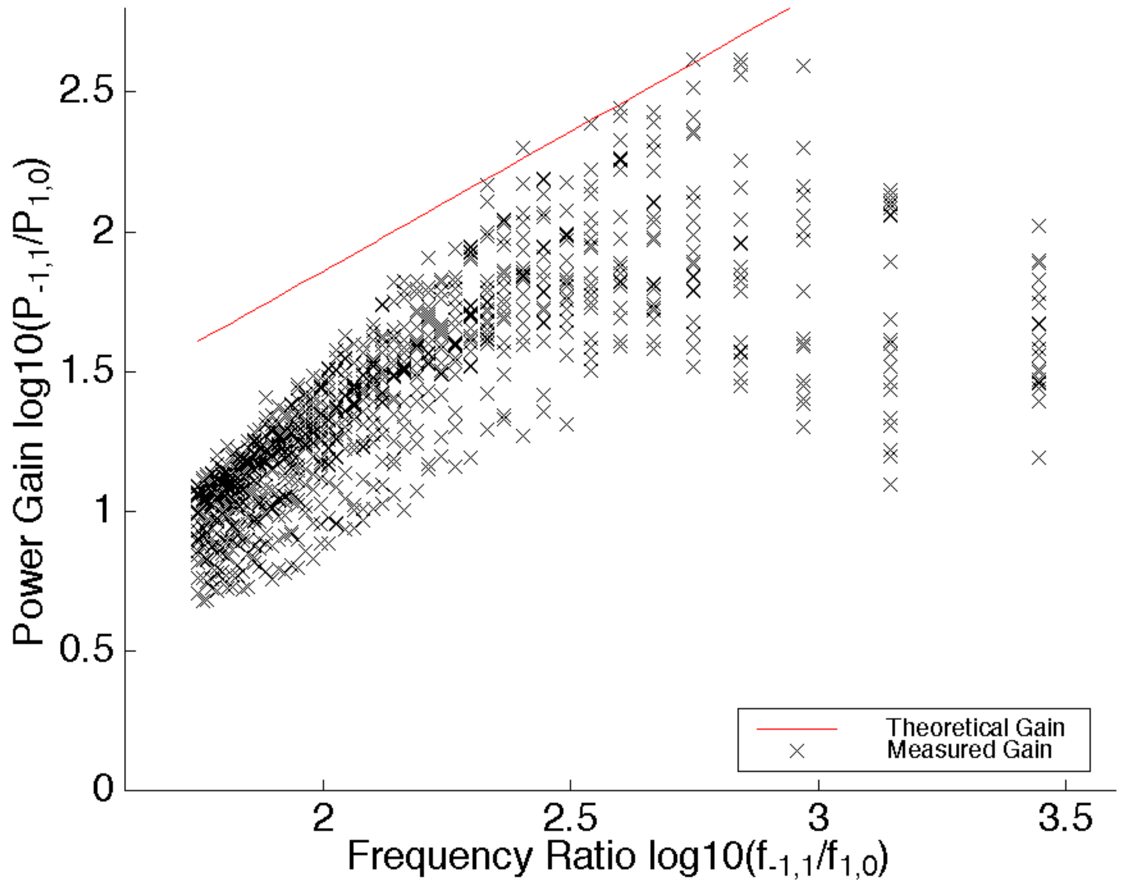
Paramp Characterization

Here, we see that we can achieve very high gain with this device under the right conditions

Pump at 5.7 kHz from 4 to 8 V p-p with a 3 V DC offset

Signal ramped from 0 to 100 Hz at 10 V p-p

Max gain is 316.2

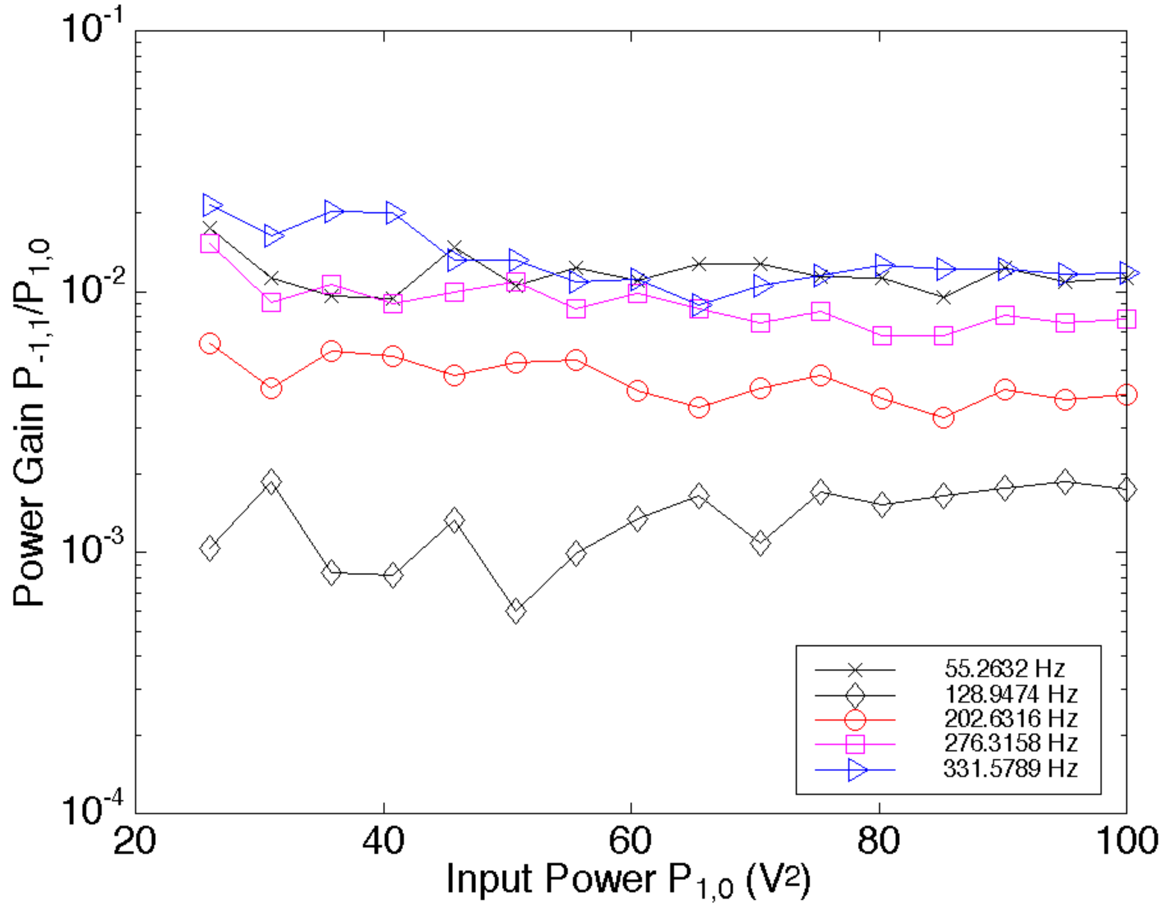


Paramp Characterization

Here, we see that we can achieve a linear response over a limited range

Pump at 10 kHz at 8 V p-p

Signal measured at 5 frequencies



Scanning Force Microscopy

There are several forces that occur in the atomic distances we are interested in:

• attractive electrostatic	SCM
• capillary (up to a few hundred Å)	
• attractive van der Waals (a few Å to a few hundred Å)	non-contact AFM
• repulsive van der Waals (up to a few Å)	contact AFM
• chemical bonds (up to a few Å)	
• magnetic	MFM

Force Gradient

All of the forces listed before are a function of tip/sample gap. This forms a force gradient.

If our mechanical system has a second-order behavior:

$$m\ddot{x} + c\dot{x} + k_{mechanical}x - F_{atomic}(x) = F_{excite}(V_e)$$

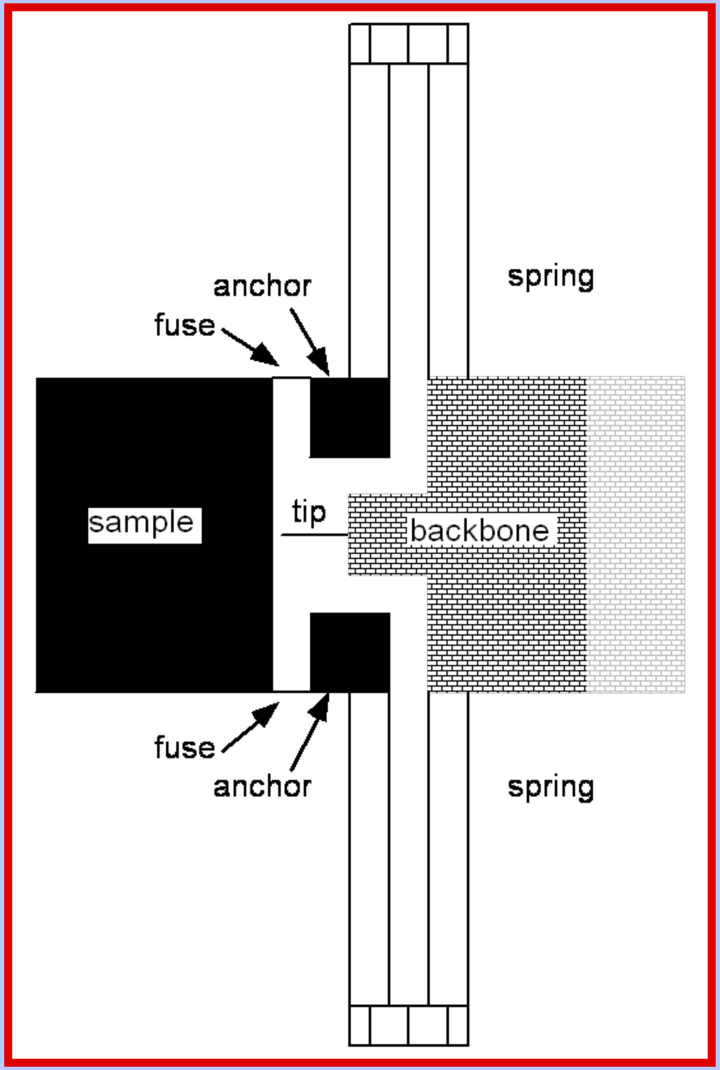
then the resonant frequency becomes:

$$f \propto \sqrt{\frac{1}{m}\left(k - \frac{\partial F_{atomic}(x)}{\partial x}\right)}$$

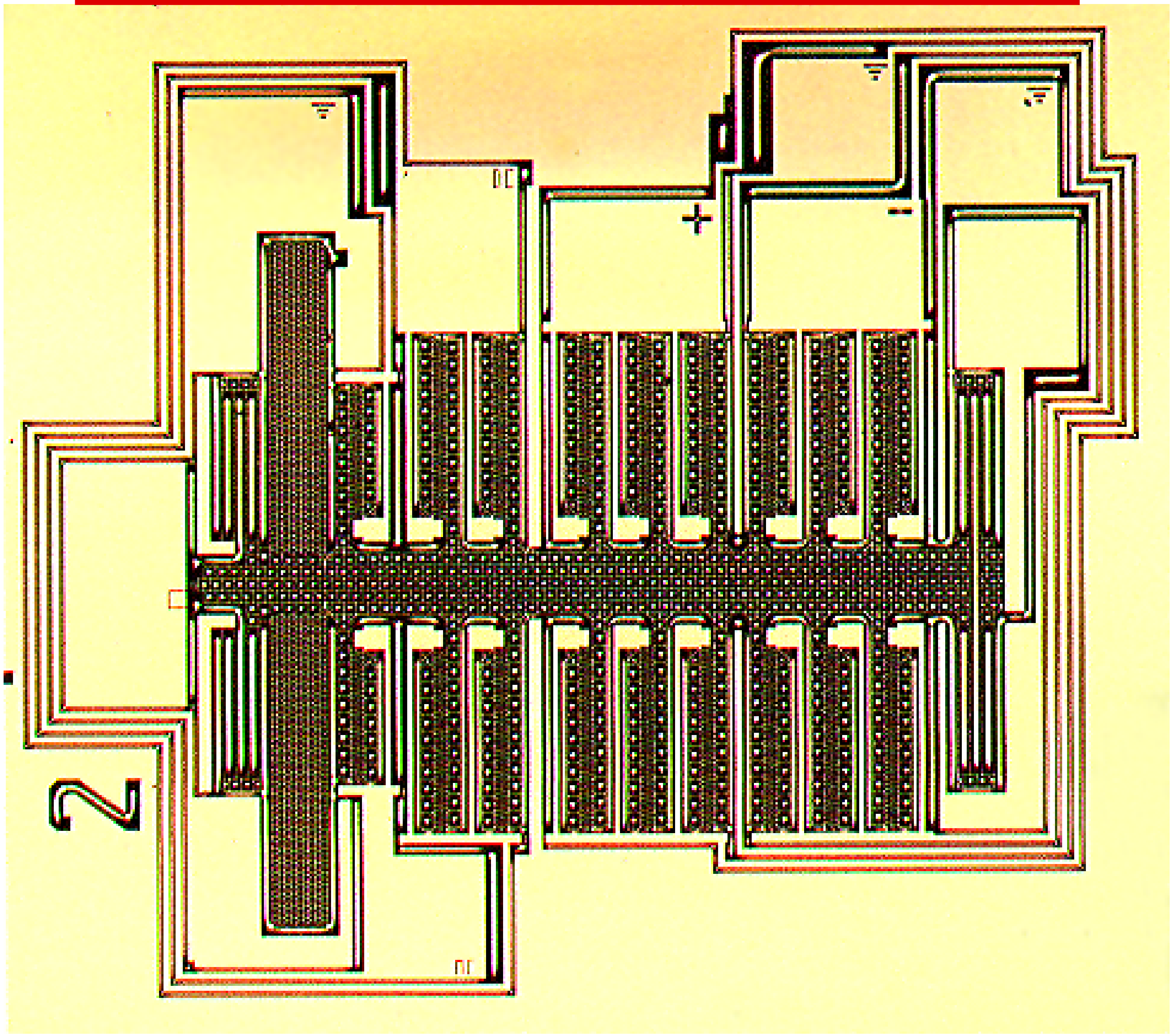
As long as the force gradient is of the same order of magnitude as the mechanical restoring force of the spring, then we will observe a frequency shift as the tip approaches the sample

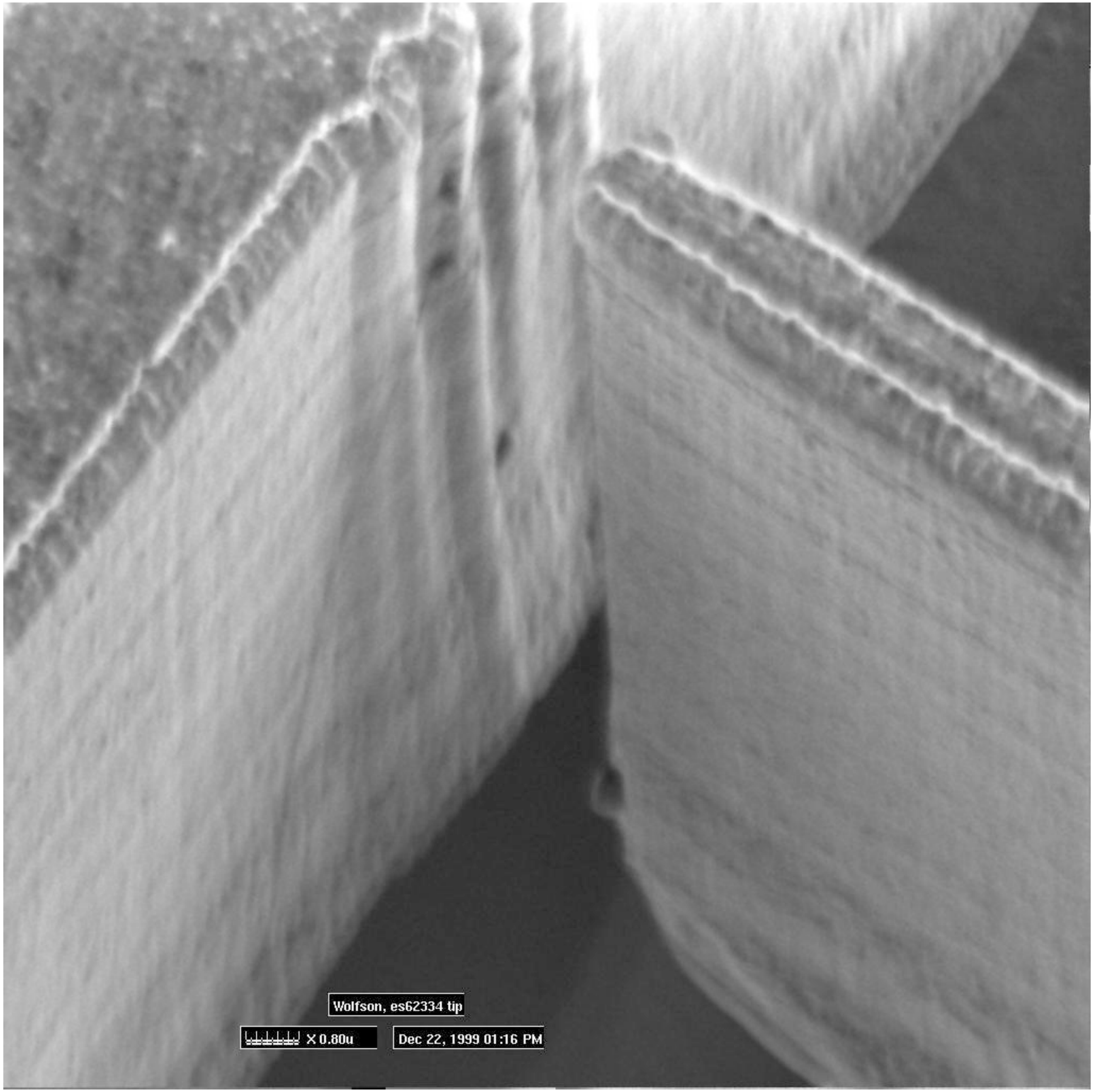
MEMS Design for AFM

For experiments with a bias on the sample, we break the fuses and the sample and tip are no longer electrically connected



MEMS Design for AFM




Wolfson, es62334 tip
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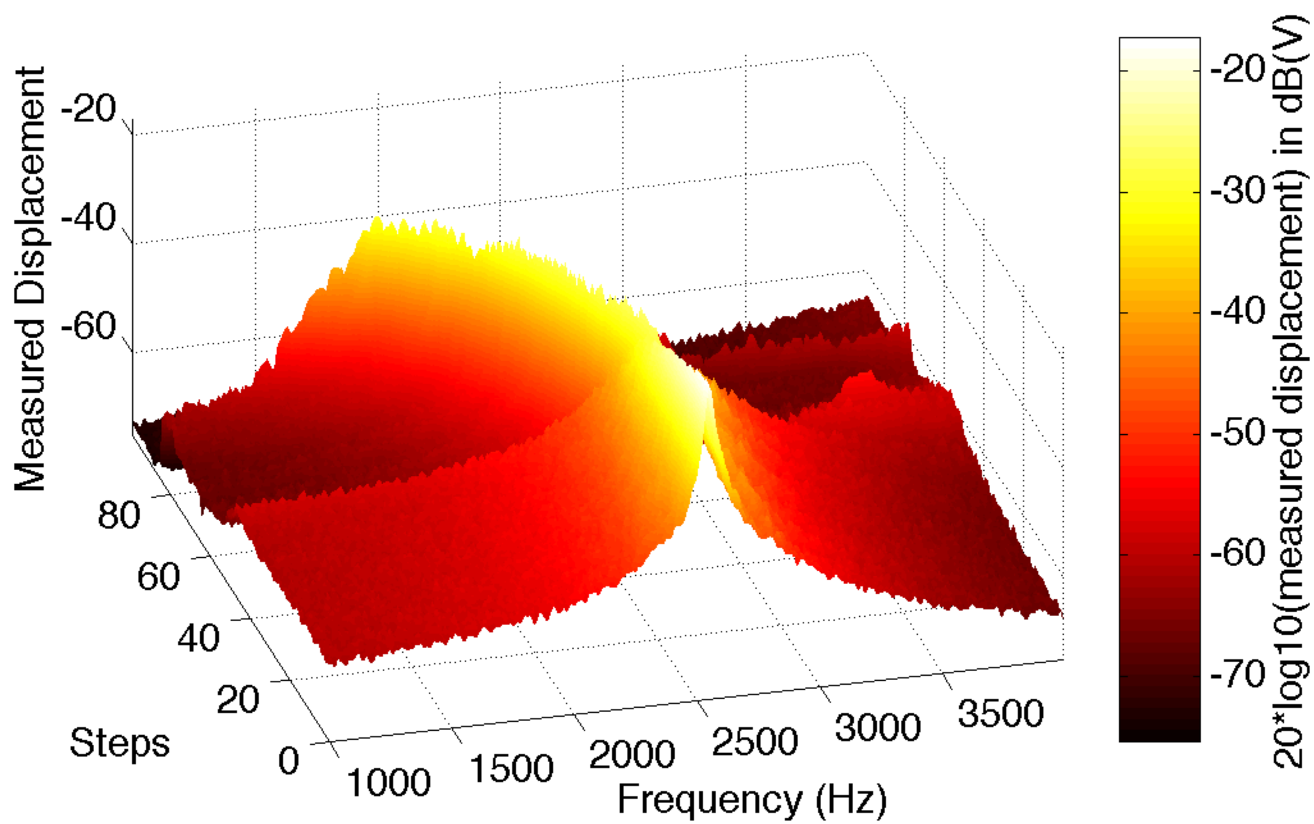
AFM Characterization

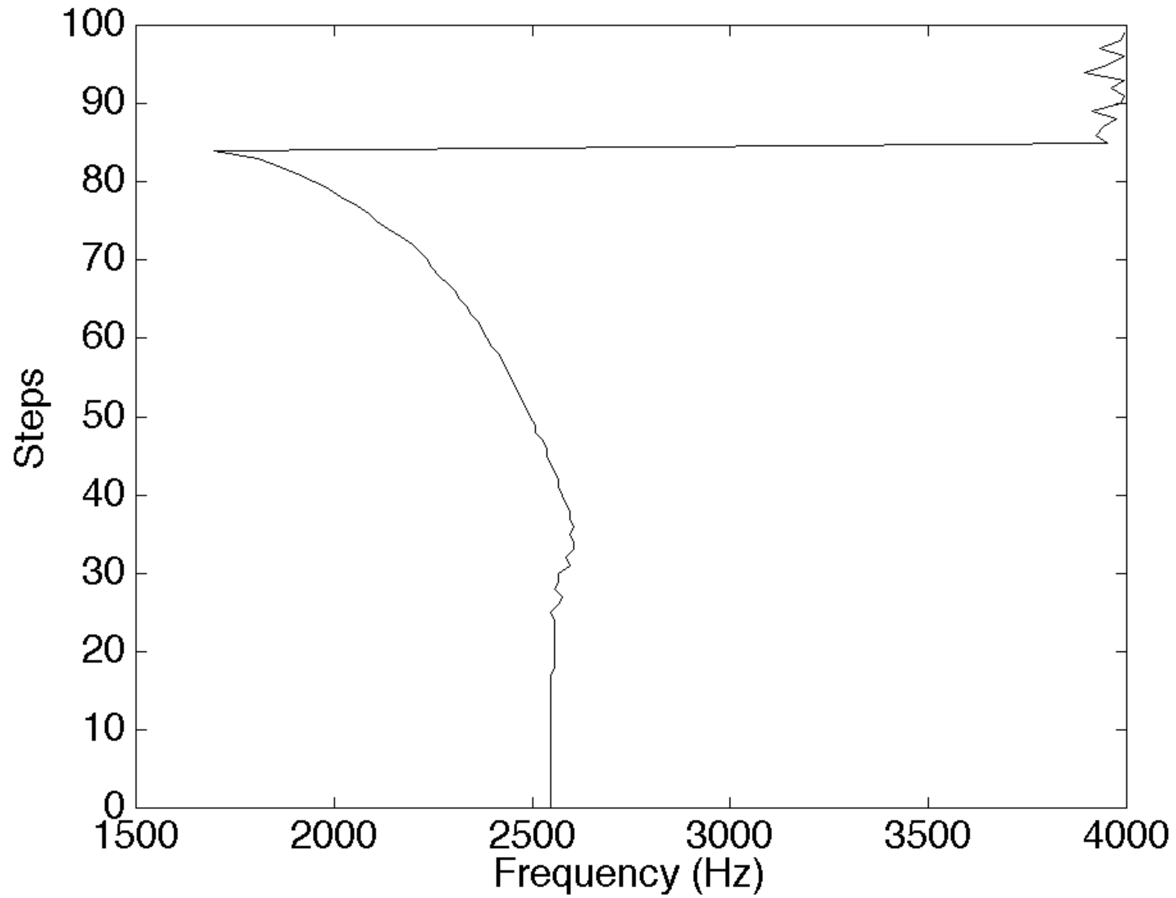
Here, we demonstrate that the resonant frequency depends on the distance between the tip and the sample.
The system is driven by a 5 V pseudo-random signal
The sample approach actuator is ramped from 16 to 19.5 V.



AFM Characterization

Here, we demonstrate that the output amplitude depends on the impedance in the system (same conditions apply as previous slide).

This trace shows the maximum amplitude at each step. This corresponds to the resonant frequency of the device at each step.

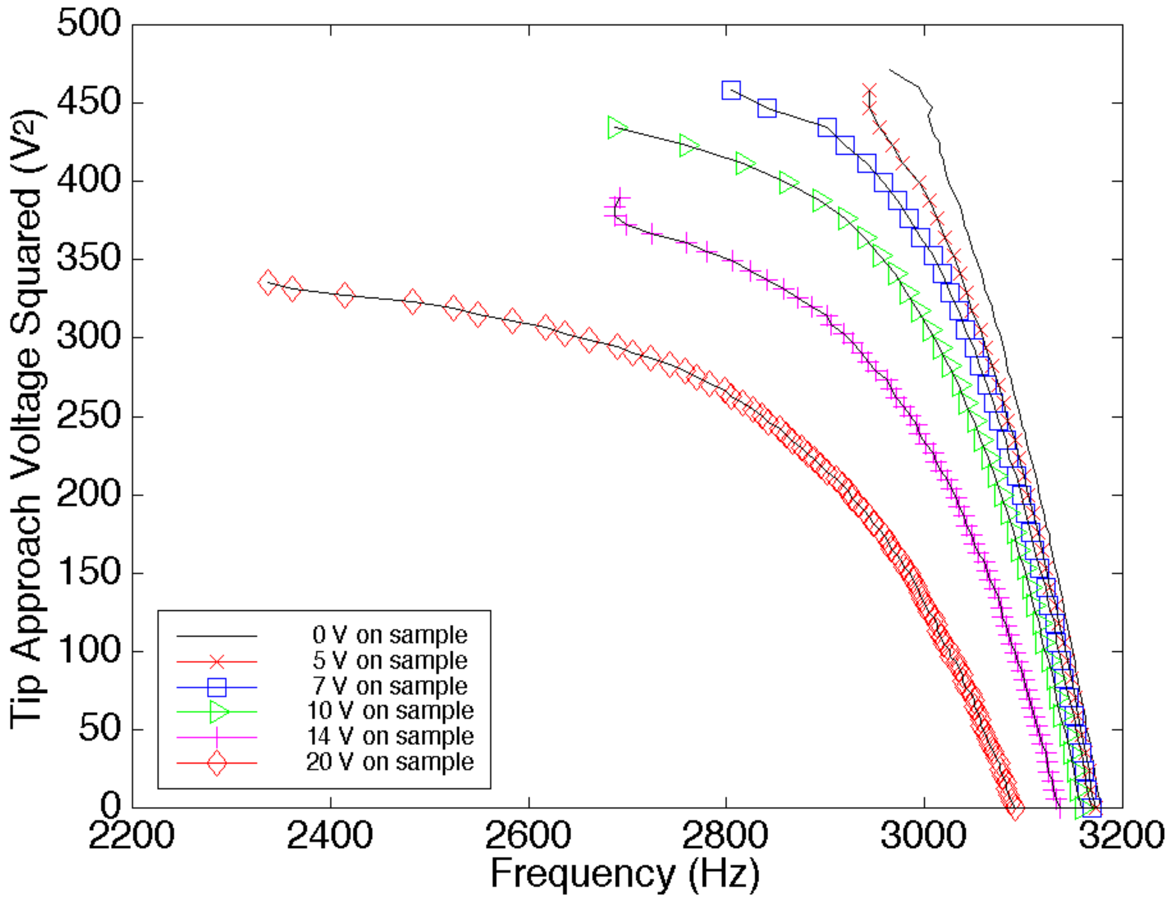


Effect of Bias on Force Gradient

Here, we see that a sample bias drastically affects the frequency response

Drive with pseudo-random signal from HP89410A at 5 V

Sample bias ranges from 0 to 20 V

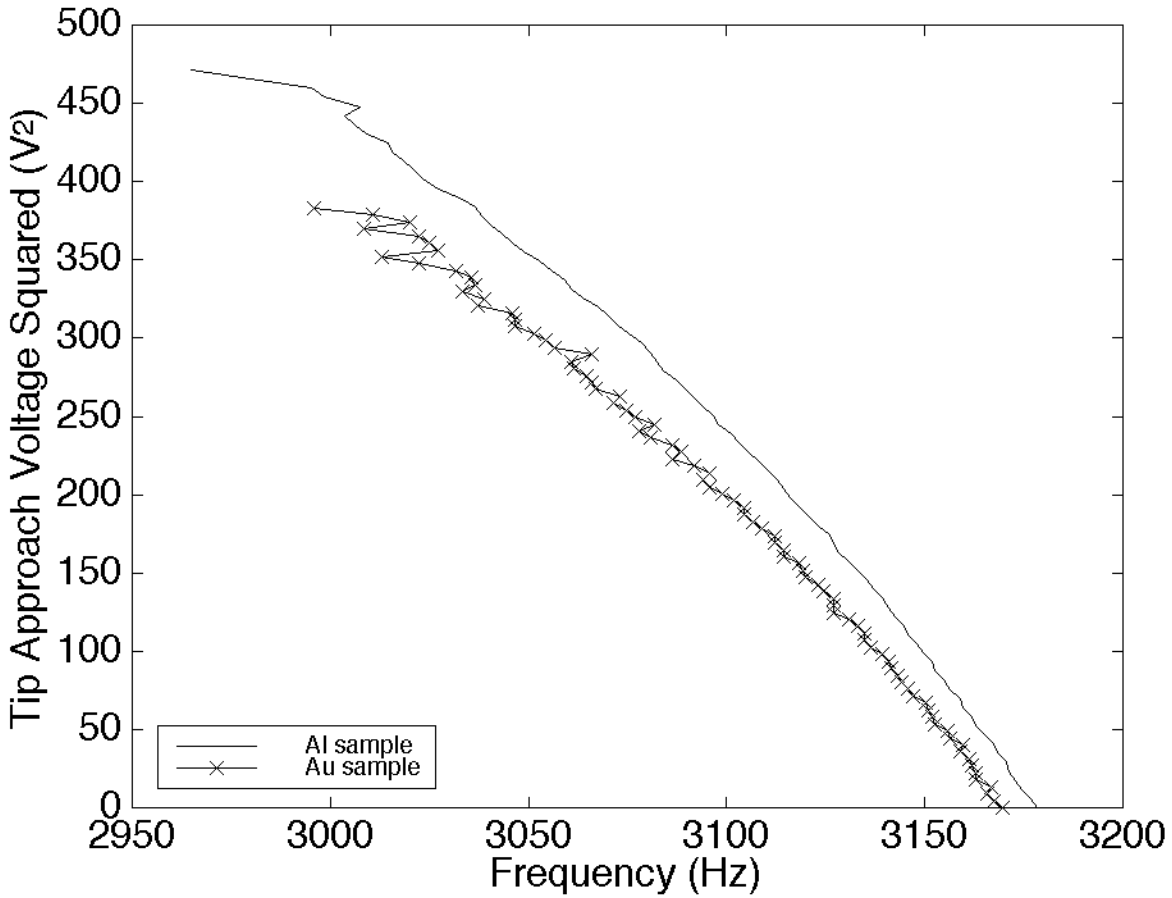


Effect of Sample Composition

Here, we have deposited 40 nm of gold on top of the original aluminum tip

Drive with pseudo-random signal from HP89410A at 5 V

The added mass is responsible for less than a 2 Hz shift

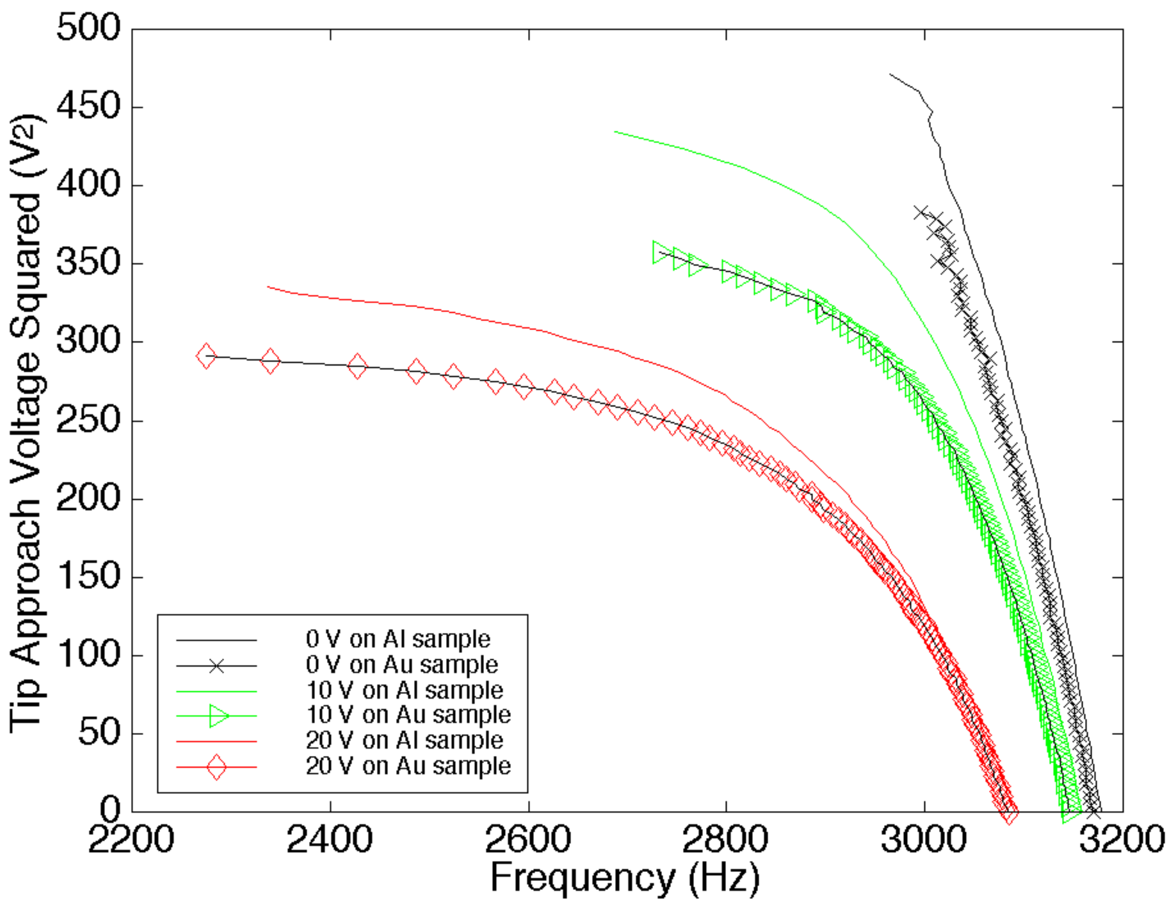


Comparison of Bias and Material

Here, we compare the change in frequency due to the addition of a bias and a change in the sample composition

Drive with pseudo-random signal from HP89410A at 5 V

Note how gold sample shifts more as the tip/sample gap decreases



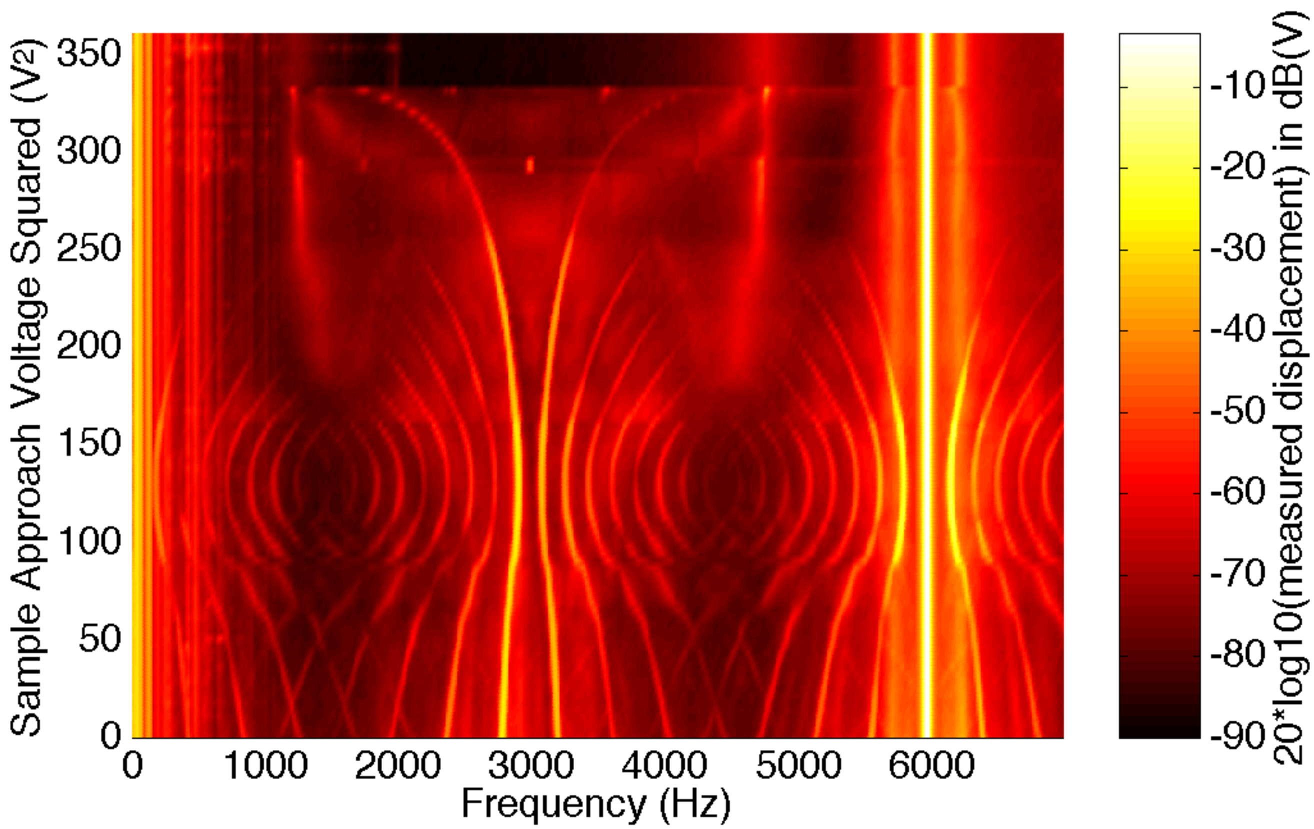
Complete System Characterization

Here, we can see all of the harmonics generated

Note how the harmonics near the pump are largest amplitude

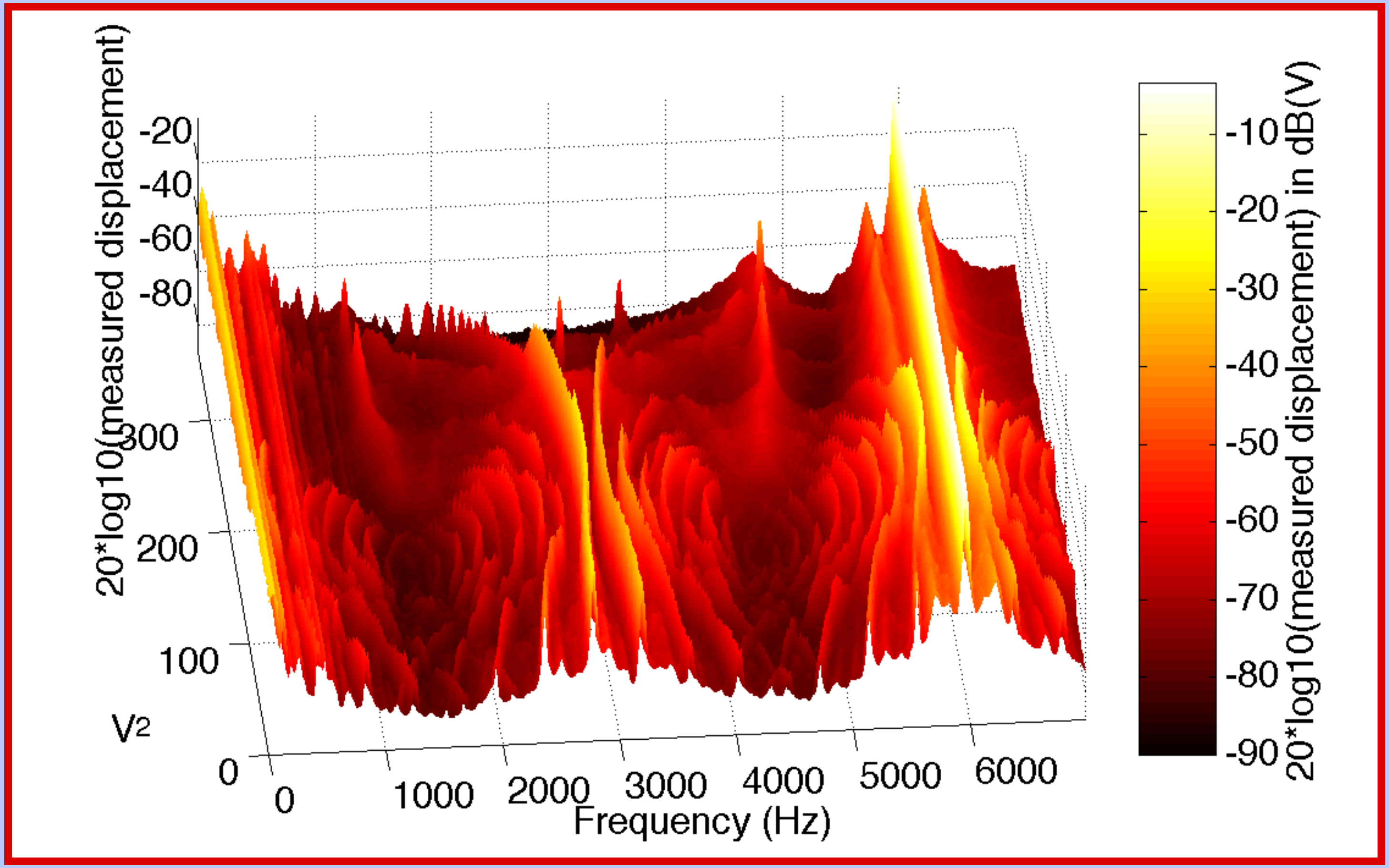
Pump is 3 V p-p with 5 V DC

Signal is pseudo-random at 5 V from 1 to 3 kHz



Complete System Characterization

We rotate the previous data to give a clearer view of the relative amplitudes



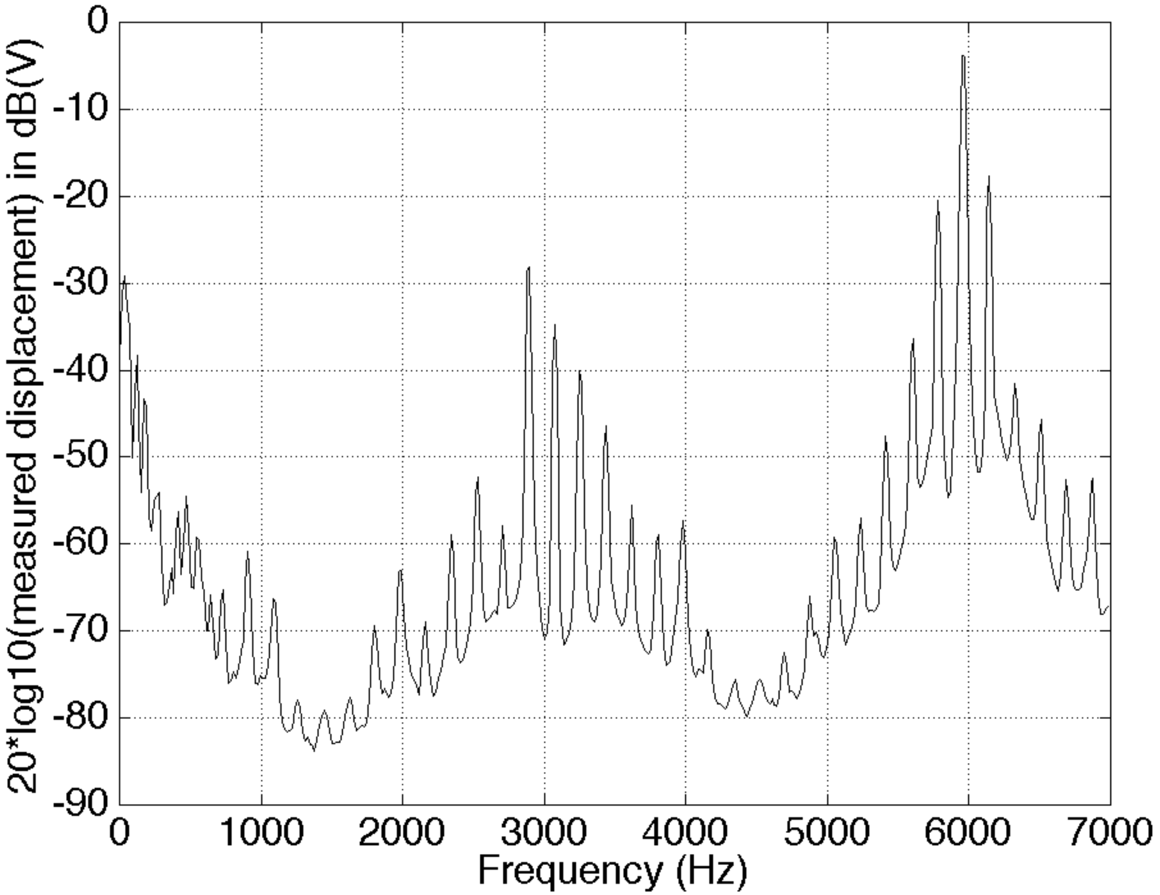
Complete System Characterization

Clearer view of gain. This is a "cross-section" at a sample approach voltage of 10.8 V

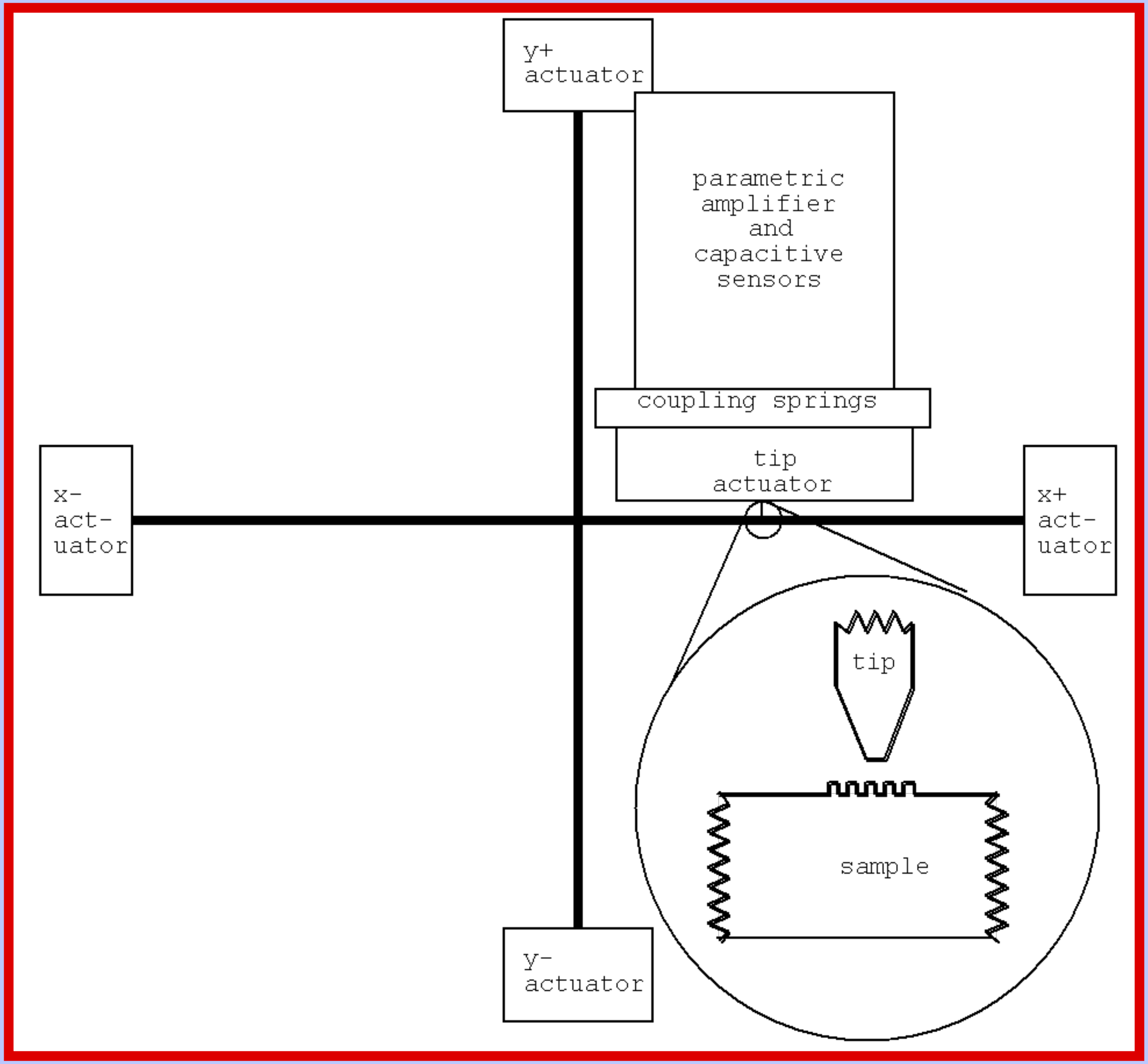
The tip is resonating at 2898 Hz at -28.1 dBV

The output signal is at 6145 Hz at -17.65 dBV

Gain is 10.45 dB.



XY Stage Schematic



Conclusion

Parametric Amplifier Subsystem

- Gain of 316.2
- Linear amplification
- Observes Manley-Rowe behavior

AFM Subsystem

- Detects van der Waals forces (AFM)
- Detects electrostatic forces (SCM)
- Integrated sample

Complete System

- Parametric amplifier mixes and amplifies results of AFM sensor
- Maximum measured gain of 244
- Fully integrated (no assembly or alignment)

Acknowledgements

This work would not be possible without the support of the following:

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- **The members of the MacDonald Research Group**
- **The CNF Staff**
- **Kionix Inc.**
- **DARPA**